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Electroweak precision constraints on vector-like fermions

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Abstract We calculate the oblique electroweak corrections and confront them with the experiments in an extension of the standard model. The new fields added are a vector-like weak doublet and a singlet fermion. After electroweak symmetry breaking there is a mixing between the components of the new fields, but there is no mixing allowed with the standard fermions. Four electroweak parameters, $\hat{S}$, $\hat{T}$, W and Y, are presented in the formalism of Barbieri et al.; these are the generalization of the Peskin–Takeuchi S, T and U. The vector-like extension is slightly constrained. $\hat{T}$ requires the new neutral fermion masses not to be very far from each other, allowing for higher mass differences for higher masses and smaller mixing. $\hat{S}$, W and Y give practically no constraints on the masses. This extension can give a positive contribution to $\hat{T}$, allowing for a heavy Higgs boson in electroweak precision tests of the standard model.

1 Introduction

Vector-like fermions appear in several extensions of the standard model (SM). They are present in extra-dimensional models with bulk fermions, see e.g. [1], in little Higgs theories [2, 3], in models of so-called improved naturalness consistent with a heavy Higgs scalar [4], in simple fermionic models of dark matter [5–8], in some dynamical models of supersymmetry breaking using gauge mediation, topcolor models [9–12], and they were also considered as the solution to the discrepancy between R_b and LEP2 measurements in the mid nineties [13, 14]. Vector-like fermions were essential ingredients in a recent proposal, in which a nontrivial condensate of new vector-like fermions breaks the electroweak symmetry and provides masses for the standard particles [15]. The potential LHC signals of vector-like quarks were discussed in [16].

Any extension of the SM must face the tremendous success of the SM in high-energy experiments. It must have evaded direct detection and fulfill the electroweak precision tests. If the scale of new physics is sufficiently high and the corrections are assumed to be universal, then the new physics only affects the finite combinations of the gauge boson self-energies. These parameters (traditionally S, T and U [17]) are constrained by experiments [18]. Barbieri et al. reconsidered the problem [19] and showed that there are actually four relevant parameters, $\hat{S}$, $\hat{T}$, W and Y, where $\hat{S}$ and $\hat{T}$ are related to the old parameters $S = 4s_W^2\hat{S}/\alpha$ and $T = \hat{T}/\alpha$. W and Y are two new parameters, and U ($\hat{U}$) is suppressed by the scale of new physics compared to T ($\hat{T}$). There are also other and more extended parameterizations known [20, 21].

In this paper we calculate the gauge-boson vacuum-polarization functions and precision electroweak observables for a simple vector-like extension of the SM, especially taking into account the mixing in the recently proposed fermion-condensate model [15]. There are earlier results for extra vector-like quarks [8, 22] and detailed calculations for the ρ parameter in the littlest Higgs model; see e.g. [23]. The new results in this paper are that we consider the mixing of new fermions belonging to different representations. We give general formulae applicable to LEP2 measurements using the four parameters of [19] and constrain the fermion-condensate model [15].

2 Extension of the standard model with vector-like fermions

We consider a simple extension of the SM based on non-chiral fermions. The new colorless fermions are an extra neutral weak $SU(2)$ singlet Ψ_S ($T = Y = 0$) and a doublet $\Psi_D = \binom{\Psi_D^+}{\Psi_D^0}$ with hypercharge 1. It is assumed that the new fermions are odd under a new Z_2 symmetry, while all the standard particles are even. This symmetry forbids mixings with standard fermions and the lightest new fermion is stable providing an ideal weakly interacting dark-matter candidate. This choice of fermions is the simplest case in which as

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a new phenomenon non-trivial mixing may appear between fermions of different representations. The mixing is essential in this model; it is responsible for symmetry breaking in [15] and it provides a simple fermionic dark-matter model (without the singlet and the mixing, the doublet is ruled out by direct dark-matter searches [5–7]).

The purely fermionic part of the new Lagrangian is

$$L_\Psi = i\overline{\Psi}_D D_\mu \gamma^\mu \Psi_D + i\overline{\Psi}_S \partial_\mu \gamma^\mu \Psi_S - m_1 \overline{\Psi}_D \Psi_D - m_2 \overline{\Psi}_S \Psi_S, \tag{1}$$

with Dirac masses m_1 and m_2. Ψ_S may have further interactions, irrelevant for our analysis. D_μ is the covariant derivative

$$D_\mu = \partial_\mu - i\frac{g}{2}\underline{\tau}\,\underline{W}_\mu - i\frac{g'}{2}B_\mu, \tag{2}$$

where $\underline{W}_\mu, B_\mu$ and g, g' are the usual weak gauge-boson fields and couplings, respectively. In a renormalizable theory including the standard Higgs doublet (H), additional Yukawa terms appear resulting in a mixing between the new neutral fermions,

$$L_{\text{Yukawa}} = \lambda_m \overline{\Psi}_D \Psi_S H + \lambda_m^* H^\dagger \overline{\Psi}_S \Psi_D. \tag{3}$$

In a version of the standard model [15], the Higgs boson is a composite state of the new fermions ($H = \overline{\Psi}_S \Psi_D$) and these Yukawa terms (and an additional contribution to the Ψ_D and Ψ_S masses) generated by condensation from effective 4-fermion interactions.

When the Higgs (or the composite operator $\overline{\Psi}_S \Psi_D$ in [15]) develops a vacuum-expectation value, $\langle H\rangle_0 = \binom{0}{v}$, with real v, non-diagonal mass terms are generated with $m_3 = (\lambda_m + \lambda_m^*)v/2$

$$L_{\text{mass}} = -m_1 \overline{\Psi}_D \Psi_D - m_2 \overline{\Psi}_S \Psi_S - m_3 \left(\overline{\Psi^0}_D \Psi_S + \overline{\Psi}_S \Psi_D^0\right). \tag{4}$$

In [15] m_1 (m_2) get contributions from the condensates. The mass matrix of the new fermions must be diagonalized via a unitary transformation to get physical mass eigenstates:

$$\begin{aligned} \Psi_1 &= c\Psi_D^0 + s\Psi_S, \\ \Psi_2 &= -s\Psi_D^0 + c\Psi_S, \end{aligned} \tag{5}$$

where $c = \cos\phi$, $s = \sin\phi$ and ϕ is the mixing angle defined by

$$2m_3 = (m_1 - m_2)\tan 2\phi. \tag{6}$$

The masses of the new neutral physical fermions Ψ_1 and Ψ_2 are $M_{1,2} = \frac{1}{2}(m_1 + m_2 \pm \frac{m_1 - m_2}{\cos 2\phi})$. The useful inverse relations are

$$\begin{aligned} m_1 &= c^2 M_1 + s^2 M_2, \\ m_2 &= s^2 M_1 + c^2 M_2. \end{aligned} \tag{7}$$

In the physical spectrum there is also a charged fermion Ψ_D^+, with mass $M_+ = m_1$ (given by (7)). In the case of an elementary scalar field λ_m is a free parameter. The mixing angle and the physical masses are basically not constrained by the theory. In [15] the gap equations determine the masses and the mixing angle. Applying further unitarity constraints, one finds that one of the neutral masses is very close to the charged mass and the mixing is rather weak [24].

The collider phenomenology and radiative corrections in the model are coming from the doublet kinetic term in (1) taking into account the mixing (5):

$$\begin{aligned} L^I &= \overline{\Psi_D^+}\gamma^\mu \Psi_D^+ \left(\frac{g'}{2}B_\mu + \frac{g}{2}W_{3\mu}\right) \\ &\quad + \left(c^2 \overline{\Psi}_1 \gamma^\mu \Psi_1 + s^2 \overline{\Psi}_2 \gamma^\mu \Psi_2 \right.\\ &\quad \left. - sc\left(\overline{\Psi}_1 \gamma^\mu \Psi_2 + \overline{\Psi}_2 \gamma^\mu \Psi_1\right)\right)\left(\frac{g'}{2}B_\mu - \frac{g}{2}W_{3\mu}\right) \\ &\quad + \left[\frac{g}{\sqrt{2}}W_\mu^+\left(c\overline{\Psi_D^+}\gamma^\mu \Psi_1 - s\overline{\Psi_D^+}\gamma^\mu \Psi_2\right) + h.c.\right]. \end{aligned} \tag{8}$$

We calculate the contribution to the electroweak precision observables from this renormalizable interaction.

3 Electroweak precision parameters

Barbieri et al. showed [19] that if the scale of new physics is sufficiently higher than the LEP2 scale and the new physics affects only the vector boson self-energies, then the most general parameterization of new physics effects uses the four parameters $\hat{S}$, $\hat{T}$, W and Y. These parameters are the generalizations of the Peskin–Takeuchi S, T and U parameters and defined by the transverse gauge-boson vacuum-polarization amplitudes:

$$\Pi_{ab}^{\mu\nu}(q^2) = g^{\mu\nu}\Pi_{ab}(q^2) + p^\mu p^\nu \text{terms}, \tag{9}$$

expanded up to quadratic order: ($ab = \{W^+W^-, W_3W_3, BB, W_3B\}$)

$$\Pi_{ab}(q^2) \simeq \Pi_{ab}(0) + q^2\Pi'_{ab}(0) + \frac{(q^2)^2}{2}\Pi''_{ab}(0) + \cdots.$$

The relevant parameters are defined by

$$(g'/g)\hat{S} = \Pi'_{W_3B}(0), \tag{10}$$

$$M_W^2 \hat{T} = \Pi_{W_3W_3}(0) - \Pi_{W^+W^-}(0), \tag{11}$$

Table 1 Global fit of the electroweak precision parameters for a light ($M_H = 115$ GeV) and a heavy ($M_H = 800$ GeV) Higgs boson

	$10^3\hat{S}$	$10^3\hat{T}$	$10^3 W$	$10^3 Y$
Light Higgs	0.0 ± 1.3	0.1 ± 0.9	0.1 ± 1.2	-0.4 ± 0.8
Heavy Higgs	-0.9 ± 1.3	2.0 ± 1.0	0.0 ± 1.2	-0.2 ± 0.8

$$2M_W^{-2} Y = \Pi''_{BB}(0), \tag{12}$$

$$2M_W^{-2} W = \Pi''_{W_3 W_3}(0), \tag{13}$$

where we use canonically normalized fields and Π functions. The form factor $\hat{T}$ has custodial and $SU_L(2)$ breaking quantum numbers, while $\hat{S}$ respects custodial symmetry and breaks $SU_L(2)$. Y and W are symmetric under both symmetries and they are important at the LEP2 energies. The result of the combined fit (excluding NuTeV) is shown in Table 1 from [19].

The calculation of the parameters is based on the general gauge-boson vacuum-polarization diagram with two non-degenerate fermions with masses m_a and m_b. We use dimensional regularization and give the result for general q^2. The coupling constants are defined in the usual manner: $L^I \sim V_\mu \bar{\Psi}(g_V\gamma^\mu + g_A\gamma_5\gamma^\mu)\Psi$. We have

$$\Pi(q^2) = \frac{1}{4\pi^2}\left(\left(g_V^2 + g_A^2\right)\tilde{\Pi}_{V+A} + \left(g_V^2 - g_A^2\right)\tilde{\Pi}_{V-A}\right), \tag{14}$$

where

$$\begin{aligned}\tilde{\Pi}_{V+A} = {} & -\frac{1}{2}\left(m_a^2 + m_b^2 - \frac{2}{3}q^2\right)\left(\mathrm{Div} + \ln\left(\frac{\mu^2}{m_a m_b}\right)\right) \\ & - \frac{(m_a^2 - m_b^2)^2}{6q^2} - \frac{1}{3}\left(m_a^2 + m_b^2\right) \\ & + \frac{5}{9}q^2 - \frac{(m_a^2 - m_b^2)^3}{12q^4}\ln\left(\frac{m_b^2}{m_a^2}\right) \\ & + \frac{1}{3}\left(\frac{(m_a^2 - m_b^2)^2}{q^2} + m_a^2 + m_b^2 - 2q^2\right) \\ & \times f\left(m_a^2, m_b^2, q^2\right)\end{aligned} \tag{15}$$

and

$$\begin{aligned}\tilde{\Pi}_{V-A} = {} & m_a m_b\left(\mathrm{Div} + \ln\left(\frac{\mu^2}{m_a m_b}\right)\right. \\ & + 2 + \frac{(m_a^2 - m_b^2)}{2q^4}\ln\left(\frac{m_b^2}{m_a^2}\right) \\ & \left. - 2f\left(m_a^2, m_b^2, q^2\right)\right).\end{aligned} \tag{16}$$

The function $f(m_a^2, m_b^2, q^2)$ is given by

$$f\left(m_a^2, m_b^2, q^2\right) = \begin{cases} \sqrt{\Delta}\,\mathrm{arctanh}\left(\frac{\sqrt{\Delta}q^2}{q^2 - (m_a + m_b)^2}\right) \\ \quad q < |m_a - m_b| \\ -\sqrt{-\Delta}\,\arctan\left(\frac{\sqrt{-\Delta}q^2}{q^2 - (m_a + m_b)^2}\right) \\ \quad |m_a - m_b| < q \text{ and } q < m_a + m_b \\ \sqrt{\Delta}\,\mathrm{arccoth}\left(\frac{\sqrt{\Delta}q^2}{q^2 - (m_a + m_b)^2}\right) \\ \quad m_a + m_b < q, \end{cases} \tag{17}$$

where we defined

$$\Delta = 1 - 2\frac{m_a^2 + m_b^2}{q^2} + \frac{(m_a^2 - m_b^2)^2}{q^4}, \tag{18}$$

and $\mathrm{Div} = 1/\epsilon + \ln 4\pi - \gamma_\epsilon$ contains the usual divergent term in dimensional regularization.

The electroweak parameters depend on the values and derivatives of the Π functions at $q^2 = 0$, and the limits are given below:

$$\begin{aligned}\tilde{\Pi}_{V+A}(0) = {} & -\frac{1}{2}\left(m_a^2 + m_b^2\right)\left(\mathrm{Div} + \ln\left(\frac{\mu^2}{m_a m_b}\right)\right) \\ & - \frac{1}{4}\left(m_a^2 + m_b^2\right) - \frac{(m_a^4 + m_b^4)}{4(m_a^2 - m_b^2)}\ln\left(\frac{m_b^2}{m_a^2}\right),\end{aligned} \tag{19}$$

$$\begin{aligned}\tilde{\Pi}_{V-A}(0) = {} & m_a m_b\left(\mathrm{Div} + \ln\left(\frac{\mu^2}{m_a m_b}\right) + 1\right. \\ & \left. + \frac{(m_a^2 + m_b^2)}{2(m_a^2 - m_b^2)}\ln\left(\frac{m_b^2}{m_a^2}\right)\right).\end{aligned} \tag{20}$$

The first and second derivatives are

$$\begin{aligned}\tilde{\Pi}'_{V+A}(0) = {} & \left(\frac{1}{3}\mathrm{Div} + \ln\left(\frac{\mu^2}{m_a m_b}\right)\right) \\ & + \frac{m_a^4 - 8m_a^2 m_b^2 + m_b^4}{9(m_a^2 - m_b^2)^2} \\ & + \frac{(m_a^2 + m_b^2)(m_a^4 - 4m_a^2 m_b^2 + m_b^4)}{6(m_a^2 - m_b^2)^3} \\ & \times \ln\left(\frac{m_b^2}{m_a^2}\right),\end{aligned} \tag{21}$$

$$\begin{aligned}\tilde{\Pi}'_{V-A}(0) = {} & m_a m_b\left(\frac{(m_a^2 + m_b^2)}{2(m_a^2 - m_b^2)} + \frac{m_a^2 m_b^2}{(m_a^2 - m_b^2)^3}\right. \\ & \left. \times \ln\left(\frac{m_b^2}{m_a^2}\right)\right),\end{aligned} \tag{22}$$

and

$$\tilde{\Pi}''_{V+A}(0) = \frac{(m_a^2+m_b^2)(m_a^4-8m_a^2m_b^2+m_b^4)}{4(m_a^2-m_b^2)^4} - \frac{3m_a^4m_b^4}{(m_a^2-m_b^2)^5}\ln\left(\frac{m_b^2}{m_a^2}\right), \tag{23}$$

$$\tilde{\Pi}''_{V-A}(0) = m_a m_b\left(\frac{(m_a^4+10m_a^2m_b^2+m_b^4)}{3(m_a^2-m_b^2)^4} + \frac{2(m_a^2+m_b^2)m_a^2m_b^2}{2(m_a^2-m_b^2)^5}\ln\left(\frac{m_b^2}{m_a^2}\right)\right). \tag{24}$$

The values of the vacuum polarizations for identical masses ($m_b = m_a$) are the smooth limits of the previous formulae and agree with direct calculation.

$$\tilde{\Pi}_{V+A}(0) = -m_a^2\,\mathrm{Div} - m_a^2\ln\left(\frac{\mu^2}{m_a^2}\right),$$

$$\tilde{\Pi}_{V-A}(0) = m_a^2\mathrm{Div} + m_a^2\ln\left(\frac{\mu^2}{m_a^2}\right),$$

$$\tilde{\Pi}'_{V+A}(0) = \frac{1}{3}\mathrm{Div} + \frac{1}{3}m_a^2\ln\left(\frac{\mu^2}{m_a^2}\right) - \frac{1}{6},$$

$$\tilde{\Pi}'_{V-A}(0) = \frac{1}{6},$$

$$\tilde{\Pi}''_{V+A}(0) = \frac{1}{10m_a^2},$$

$$\tilde{\Pi}''_{V-A}(0) = \frac{1}{30m_a^2}.$$

The new vector-like fermions contribute to the complete vacuum polarization as the sum of (15) and (16). We define

$$\tilde{\Pi}_V(m_a, m_b, q^2) = \tilde{\Pi}_{V+A}(m_a, m_b, q^2) + \tilde{\Pi}_{V-A}(m_a, m_b, q^2). \tag{25}$$

In the following the index V is omitted and we use $\tilde{\Pi} = \tilde{\Pi}_V$.

The $\hat{S}$ parameter (10) is then given by

$$\hat{S} = \frac{g^2}{16\pi^2}\big(+\tilde{\Pi}'(M_+, M_+, 0) - c^4\tilde{\Pi}'(M_1, M_1, 0) - s^4\tilde{\Pi}'(M_2, M_2, 0) - 2s^2c^2\tilde{\Pi}'(M_2, M_1, 0)\big). \tag{26}$$

The first three terms cancel the divergent contribution of the last one.

The $\hat{T}$ parameter (11), related to $\Delta\rho$, is also finite,

$$\hat{T} = \frac{g^2}{M_W^2 16\pi^2}\big[+\tilde{\Pi}(M_+, M_+, 0) + c^4\tilde{\Pi}(M_1, M_1, 0) + s^4\tilde{\Pi}(M_2, M_2, 0) + 2s^2c^2\tilde{\Pi}(M_2, M_1, 0) - 2c^2\tilde{\Pi}(M_+, M_1, 0) - 2s^2\tilde{\Pi}(M_+, M_2, 0)\big]. \tag{27}$$

The Y and the W parameters differ only in the coupling constants

$$Y = M_W^2\frac{g'^2}{32\pi^2}\big[\tilde{\Pi}''(M_+, M_+, 0) + c^4\tilde{\Pi}''(M_1, M_1, 0) + s^4\tilde{\Pi}''(M_2, M_2, 0) + 2s^2c^2\tilde{\Pi}''(M_2, M_1, 0)\big], \tag{28}$$

$$W = M_W^2\frac{g^2}{32\pi^2}\big[\tilde{\Pi}''(M_+, M_+, 0) + c^4\tilde{\Pi}''(M_1, M_1, 0) + s^4\tilde{\Pi}''(M_2, M_2, 0) + 2s^2c^2\tilde{\Pi}''(M_2, M_1, 0)\big]. \tag{29}$$

The first three terms in the parentheses give $W = \frac{g^2}{240\pi^2}M_W^2 \times (1/M_+^2 + c^4/M_1^2 + s^4/M_2^2)$ in agreement with [25], taking into account that the authors of this reference considered Majorana fermions. The last term is of the same order of magnitude in $M_W/M_{\{1,2,+\}}$. Here W and Y are always non-negative, fulfilling the positivity constraints proven in [26].

4 Numerical results

There are three free parameters in the model to confront with experiment: the two neutral masses ($M_{1,2}$) and the mixing angle ϕ; we have $s^2 = \sin^2\phi$ and $c^2 = \cos^2\phi$. The mass of the charged fermion is given by $M_+ = c^2M_1 + s^2M_2$; see (7). If there is non-negligible mixing between the singlet and the doublet the new particles are expected to be heavier than approximately 100 GeV from LEP1 and LEP2, as they have ordinary couplings with the gauge bosons. Without considerable mixing the singlet fermion could avoid the LEP bound but the mixing is essential in the model for symmetry breaking in [15] and also for providing good dark-matter candidates, which are the two main motivations of the proposal. For relatively light new particles (with masses 100–150 GeV), the oblique parameters give a rough estimate of the radiative corrections [25]. Replacing $M_1 \leftrightarrow M_2$ and $c^2 \leftrightarrow s^2 = 1 - c^2$ gives the same oblique parameters. If there is no real mixing, $c^2 = 0$ or 1; or if $M_1 = M_2 = M_+$, then there is one degenerate vector-like fermion doublet and a decoupled singlet, and $\hat{S}$ and $\hat{T}$ vanish explicitly. In this case the new sector does not violate $SU_L(2)$ and there is an exact custodial symmetry. Increasing the mass difference in the remnants of the original doublet by increasing the $|M_1 - M_2|$ mass difference and/or moving away from the non-mixing case, $c^2 = 0$, or 1, results in increasing $\hat{S}$ and $\hat{T}$. For a small violation of the symmetries, $\hat{S}$ and $\hat{T}$ are expected to be small.

The $\hat{S}$, W and Y parameters are small for masses in the range from 100 GeV up to few TeV; the only exception is

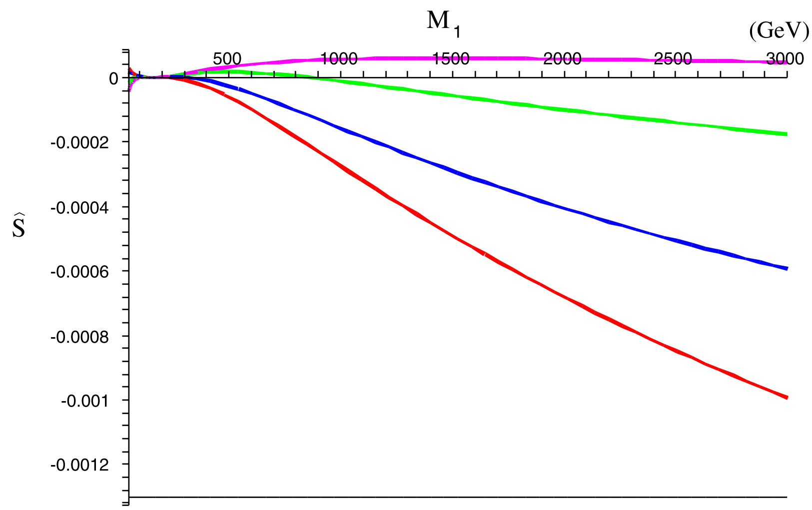

Fig. 1 The $\hat{S}$ parameter versus M_1 for $M_2 = 150$ GeV for $c^2 = 0.2, 0.4, 0.6, 0.8$, respectively, from the bottom upwards (from *red* to *magenta*); the *horizontal line* is the 1σ experimental lower bound

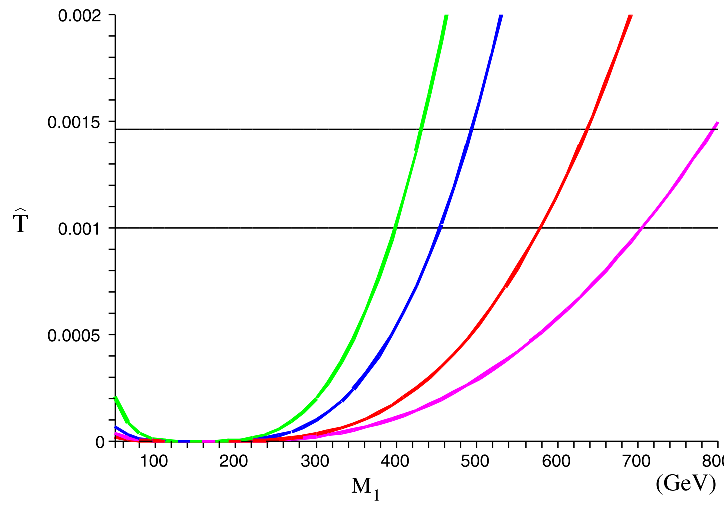

Fig. 2 The $\hat{T}$ parameter versus M_1 for $M_2 = 150$ GeV for $c^2 = 0.9, 0.1, 0.2, 0.55$ from the bottom upwards; the *horizontal lines* are the 1σ and 1.6σ experimental upper bounds

$\hat{T}\,(T)$, which is sensitive to the mass differences. These features were predicted using simple assumptions in [24]. We discuss in detail the case of a light Higgs boson (Table 1).

Generally the $\hat{S}\,(S)$ parameter depends only on the masses of the new particles and the mixing angle. It contains no further dimensional parameter. For reasonable masses (below a few TeV) it is always in agreement with the 1σ experimental bounds for $M_{1,2} \geq 100$ GeV. See Fig. 1. For higher masses $|\hat{S}|(|S|)$ is even smaller. S can have both signs.

The $\hat{T}$ parameter (27) is more sensitive to the value of $M_{1,2}$. The constraints are the strongest for $c^2 \simeq 0.56$; below and above this mixing the absolute value of $\hat{T}$ decreases and gives weaker bounds on the masses. The mass difference of the new fermions must not exceed a critical value, $|M_1 - M_2| \leq 250$ (400) for the mass of the lighter fermion 150 (500) GeV for $c^2 \simeq 0.56$. For small mixing (c^2 close to 0 or 1) there are very weak or simply no constraints. Figure 2 shows as an example $\hat{T}$ for $M_2 = 150$ GeV as a function of M_1 for various mixings. $c^2 = 0\,(1)$ gives a horizontal line, $\hat{T} = 0$. The maximum value of M_2 versus M_1 is given by the upper three continuous and dotted lines for various c^2 in Fig. 4. $\hat{T}\,(T)$ is always positive allowing for a heavy Higgs particle.

The W parameter is sensitive to the ratio M_W^2/M_i^2, $i = 1, 2, +$. It is largest for relatively small masses approximately (150 GeV), but W is still well within the 1σ experimental limits. For higher masses W is even smaller. See Fig. 3. The Y parameter is the same function of the masses and mixing angles as W. The smaller gauge-coupling multiplier provides weaker constraints.

If the Higgs boson is heavy, e.g. $M_H = 800$ GeV (see Table 1) the central value of $\hat{S}$ decreases and $\hat{T}$ increases compared to the light Higgs case. $\hat{S}$ and W give practically no constraints. Increasing the Higgs mass the standard model moves away in the (S, T) plane from the experimentally allowed ellipse [27]. The negative contribution of the heavy Higgs to the $\hat{T}$ parameter can be compensated by the positive $\hat{T}$ contribution of the new fermions with a considerable mass difference; for example (150, 400) GeV or

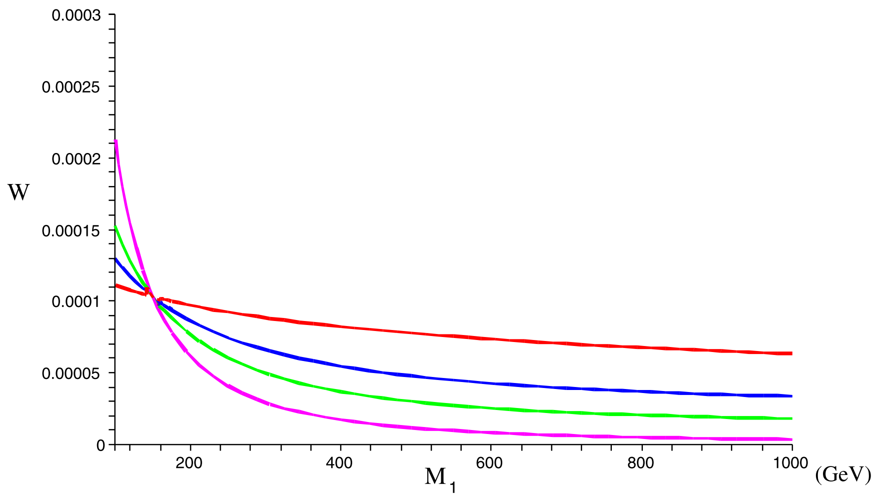

Fig. 3 The W parameter versus M_1 for $M_2 = 150$ GeV for $c^2 = 0.1, 0.3, 0.5, 0.9$ respectively from top downwards at high M_1, the 1σ experimental bound is at 0.0013, outside the figure

(500, 900) GeV and a large mixing, $c^2 \sim 0.5$. Non-degenerate vector-like fermions with reasonable mixing allow for a space for heavy Higgs in the precision tests of the standard model.

5 Electroweak precision constraints on the fermion-condensate model

In what follows we apply the previous calculations to an effective model of electroweak symmetry breaking. In the fermion-condensate model [15] the Higgs boson in (3) is a composite state of the new fermions. Gap equations were derived and solved for the parameters of the model. Applying further perturbative unitarity arguments constrains the model seriously [24]. The fermion-condensate model is an effective model with a natural cutoff. Though the whole model is non-renormalizable the sector of the model considered in our calculation is renormalizable; the divergencies are cancelled in the physical observables calculated either by dimensional regularization or by a momentum cutoff. Therefore the one-loop calculations presented here give good estimates of the electroweak observables in the fermion-condensate model for masses reasonably smaller than the cutoff.

Without loss of generality we can choose M_1 to be the mass of the lighter neutral fermion.

A summary of the results of [24] now follows.

For a cutoff $\Lambda = 3$ TeV, we get $M_1 \leq 230$ GeV; the charged fermion mass is between the neutral ones, $M_+ = c^2 M_1 + s^2 M_2$. Decreasing c^2, the charged and the heavier neutral fermions become more and more degenerate. There is a maximum value for c^2 from unitarity for each M_1: it is $c^2 < 0.27$ for $M_1 = 100$ GeV and gets smaller for higher masses, 0.19 for 150 GeV and 0.09 for 200 GeV, and reaches 0 for $M_1 \simeq 230$ GeV, providing the upper bound for M_1. There is a lower bound on M_2 depending on M_1 given by

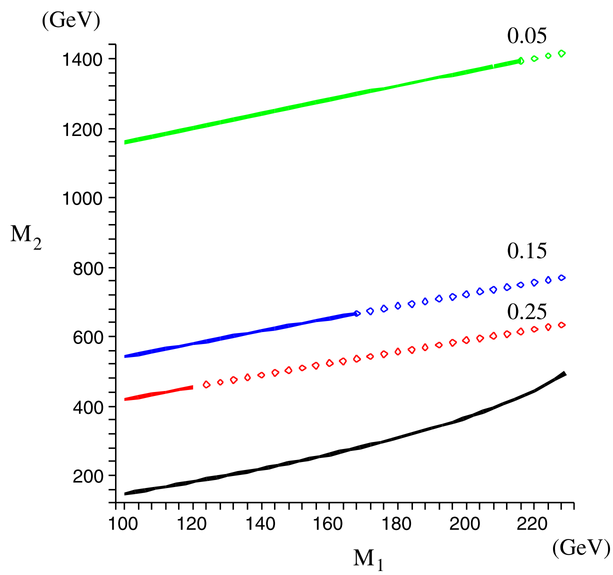

Fig. 4 The lower bound on M_1 and M_2 from the gap equation and unitarity (*black curve*) and upper bound for various mixing angles, $c^2 = 0.05, 0.15, 0.25$ from top downwards from T parameter at 90% C.L. The *dotted* part of the curves are constraints from the gap equations not fulfilling unitarity (these values of c^2 are not accessible for the given M_1) and give no upper bound

the extreme $M_+ \simeq M_2(c^2 = 0)$ case $M_2 > 145$–490 GeV for M_1 varying from 100 to 230 GeV; see the lowest black curve in Fig. 4.

The solutions of the gap equations are in agreement with the experimental constraints on $\hat{S}$ and W. As we have seen in the general case the $\hat{T}$ parameter (Fig. 2) provides an upper bound on the mass difference of the neutral particles depending on the value of c^2, the cosine of the mixing angle. In the fermion-condensate model c^2 is constrained by the gap equation (<0.27). Figure 4 shows the upper bound on M_2 for different c^2 versus M_1 and the lower bound from the gap equations and unitarity. The allowed range is above the lowest (black) curve and below the upper three curves depending on c^2, the upper limit disappears for $c^2 = 0$ when $M_+ = M_2$ and there is no real mixing.

The calculation presented in this paper shows that the fermion-condensate model is less constrained than assumed by the naive estimates in [24]. The formulae derived here can be applied not just to [15], but to various models generating the Lagrangian (3).

6 Conclusions

We have calculated the oblique corrections in an extension of the standard model based on vector-like weak singlet and doublet fermions. Due to the non-diagonal mass terms (4) mixing occurs between the singlet and the neutral component of the doublet, which breaks the electroweak symmetry. The oblique corrections were presented in the formalism of Barbieri et al. [19]. There are four relevant parameters, $\hat{S}$, $\hat{T}$, W and Y, and they are indeed in of the same order of magnitude in the allowed mass range, as expected. Y is the same function of the masses and mixing angle as W with a smaller coupling constant, but with weaker constraints; therefore, we kept $\hat{S}$, $\hat{T}$ and W. The corrections depend on the new fermion masses ($M_{1,2}$) and the mixing angle. The $\hat{S}$ and W parameters are always in agreement with experiment for masses below a few TeV. The $\hat{T}$ (T) parameter measures the custodial symmetry breaking; the custodial symmetry is exact in the new sector, if there is no physical mixing: $c^2 = 0, 1$ or $M_1 = M_2$. Depending on the mixing angle it allows in the most stringent case for $c^2 \simeq 0.56$ and a maximal mass difference, $|M_1 - M_2| \lesssim 250$ GeV at 1σ for relatively small lighter neutral mass ($\sim$150 GeV). A higher mass difference is allowed for higher $M_{1,2}$ masses or different mixing. This extension/modification nicely accommodates a heavy Higgs in the standard model. The lightest new fermion is stable and a good dark-matter candidate. The model can be tested at LHC in a Drell–Yan process [15] or via jetmass analysis [16]. Nearing the completion of our work we received a preprint which deals with a similar topic, but with a different fermion representation, approach and mixing allowed with the standard fermions [28].

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