NOTE ON UNITARITY CONSTRAINTS IN A MODEL FOR A SINGLET SCALAR DARK MATTER CANDIDATE

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We investigate perturbative unitarity constraints in a model for a singlet scalar dark matter candidate. Considering elastic two particle scattering processes of the Higgs particle and the dark matter candidate, a real Klein–Gordon scalar field, perturbative unitarity constrains the self-couplings of the scalar fields.

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Despite the unique success of the Standard Model (SM) in the particle physics accelerator experiments, there have been strong indications of fundamental physics related to particle physics beyond the SM in the past few years, such as neutrino oscillations and masses, the baryon asymmetry of the Universe cannot be explained by the SM, recent fits to the cosmological parameters [1] need dark energy. About 23% of the total energy density of the Universe is made up of some dark matter. Assuming that gravity does not change significantly at distances larger than a few kpc the dark matter must be non-baryonic to maintain the success of big bang nucleosynthesis.

These problems were addressed among others in [2] where a possible minimal extension of the SM was proposed. They added the minimal number (6) of new degrees of freedom purely to answer the empirical challenges. The non-baryonic dark matter candidate is assumed to be a Z_2 symmetric gauge singlet scalar field, S. It can account for the observed dark matter abundance and is consistent with the limit from CDMS-II experiment [3].

Davoudiasl *et al.*, [2] considered a few consequences of the model such as triviality and stability of the Higgs potential, Higgs decays into new particles.

In this note we apply perturbative unitarity [4] to a singlet dark matter field S coupled to the Higgs field H and itself in the SM completed by terms describing S, H interactions. The constraints are valid also for [2] because the inflaton field of [2] is too heavy to participate in the scattering. We get various upper bounds for the relevant scalar couplings.

Start with the minimal renormalizable extension of the Standard Model providing a scalar non-baryonic dark matter candidate S. S is required to be odd under a Z_2 symmetry in order to be stable. The odd singlet scalar can have renormalizable interaction only with the standard Higgs and not with the ordinary fermions and gauge bosons. This model was proposed earlier in the literature by Silveira and Zee [5], a complex scalar field case was studied in [6]. The parameters of the model were first constrained by Burgess *et al.* [7, 8], later in [2].

The Lagrangian of the scalar sector is

$$L_{SH} = |D_\mu H|^2 - \frac{\lambda}{2}\left|H^\dagger H - \frac{v^2}{2}\right| + \frac{1}{2}\partial_\mu S\partial^\mu S - \frac{1}{2}m_0^2 S^2 - \frac{k}{2}H^\dagger H\, S^2 - \frac{\lambda_S}{4!}S^4. \quad (1)$$

Beside the usual Higgs potential parameters $\lambda, v = 254$ GeV, there are three new parameters m_0, k, λ_S determining the properties of S. The potential of the H–S sector is bounded from below if $\lambda, \lambda_S > 0$ and $k > 0$ or

$$3k^2 < \lambda_S \lambda\,, \qquad \text{for } k < 0\,. \quad (2)$$

The Higgs field gets a vacuum expectation value v while $\langle S\rangle = 0$ in order to respect the Z_2 symmetry. The Higgs mechanism generates a mass of $m_H^2 = \lambda v^2$ for the Higgs and also contributes to the mass of the S particle

$$m_S^2 = m_0^2 + \frac{1}{2}kv^2\,. \quad (3)$$

$m_s^2 > 0$ is required for $\langle H\rangle = (0, v/\sqrt{2})$ and $\langle S\rangle = 0$ be a local minimum. This is also a global minimum as long as $m_0^2 > -\frac{1}{2}v^2\sqrt{\frac{1}{3}\lambda\lambda_S}$ [8]. After electroweak symmetry breaking in the unitarity gauge the potential of the scalar sector becomes

$$\begin{aligned} V_{SH} \;=\; & \frac{\lambda}{8}H^4 + \frac{\lambda}{2}H^3 v + \frac{\lambda}{2}H^2 v^2 + \frac{1}{2}\left(m_0^2 + \frac{1}{2}kv^2\right)S^2 \\ & + \frac{k}{2}vHS^2 + \frac{k}{4}H^2S^2 + \frac{\lambda_S}{4!}S^4\,. \end{aligned} \quad (4)$$

Next we apply tree-level perturbative unitarity [4] to scalar elastic scattering processes in the model (4). The zeroth partial wave amplitude

$$a_0 = \frac{1}{32\pi}\sqrt{\frac{4p_f^{\mathrm{CM}}p_i^{\mathrm{CM}}}{s}}\int_{-1}^{+1} T_{2\to 2}\, d\cos\theta \tag{5}$$

must satisfy $|\mathrm{Re}\, a_0| \leq \frac{1}{2}$. s is the centre of mass (CM) energy and $p_{i,f}^{\mathrm{CM}}$ are the initial and final momenta in CM system.

There are three possible two particle states HH, HS, SS and four scattering processes. Inclusion of the gauge bosons does not significantly alter our consideration since they do not interact with the new singlet scalar S at tree level.

(1) $HH \to HH$. The tree-graphs contributing to this process are drawn in Fig. 1. We get

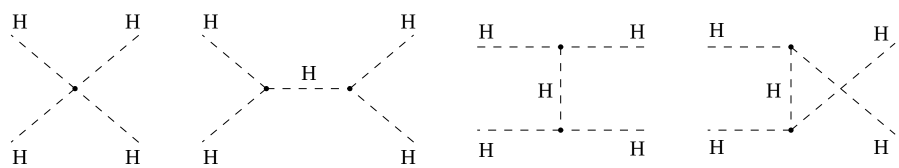

Fig. 1. Tree level Feynman diagrams for $HH \to HH$ in the SM.

$$T_{HH\to HH} = 3\frac{m_H^2}{v^2}\left(1 + 3m_H^2\left(\frac{1}{s-m_H^2} + \frac{1}{t-m_H^2} + \frac{1}{u-m_H^2}\right)\right). \tag{6}$$

(2) $SS \to SS$. The contact graph and the H-exchange graphs can be seen in Fig. 2.

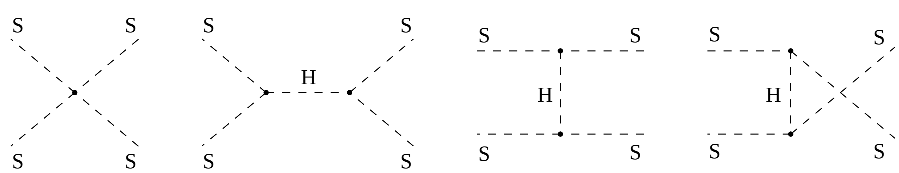

Fig. 2. Tree level Feynman diagrams for $SS \to SS$.

$$T_{SS\to SS} = \lambda_S + k\left(\frac{kv^2}{s-m_H^2} + \frac{kv^2}{t-m_H^2} + \frac{kv^2}{u-m_H^2}\right). \tag{7}$$

(3) $SS \to HH$. The T matrix (Fig. 3) is

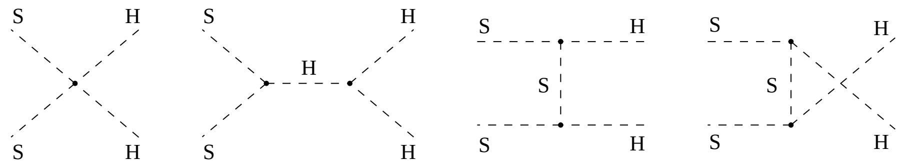

Fig. 3. Tree level Feynman diagrams for $SS \to HH$.

$$T_{SS\to HH} = k\left(1 + 3m_H^2 \frac{1}{s - m_H^2} + kv^2\left(\frac{1}{t - m_S^2} + \frac{1}{u - m_S^2}\right)\right). \qquad (8)$$

(4) $HS \to HS$. The T matrix (Fig. 4.) is

$$T_{HS\to HS} = k\left(1 + v^2\left(\frac{k}{s - m_S^2} + \frac{3\lambda}{t - m_H^2} + \frac{k}{u - m_S^2}\right)\right). \qquad (9)$$

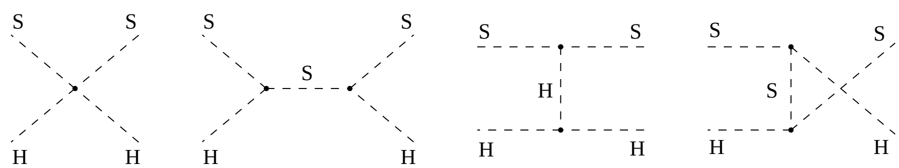

Fig. 4. Tree level Feynman diagrams for $SH \to SH$.

From (5) we have the following partial wave projection of the coupled system in the $J = 0$ channel

$$a_0(HH \to HH) = \frac{3m_H^2}{16\pi v^2}\sqrt{1 - \frac{4m_H^2}{s}}\left(1 + \frac{3m_H^2}{s - m_H^2} - \frac{6m_H^2}{s - 4m_H^2}\ln\left(\frac{s}{m_H^2} - 3\right)\right),$$

$$a_0(SS \to SS) = \frac{1}{16\pi}\sqrt{1 - \frac{4m_S^2}{s}}\left(\lambda_S + \frac{k^2v^2}{s - m_H^2} - \frac{2k^2v^2}{s - 4m_S^2}\ln\left(\frac{s - 4m_S^2}{m_H^2} + 1\right)\right),$$

$$a_0(SS\to HH) = \frac{k}{16\pi}\kappa_{SS\to HH}\left(1+3m_H^2\left(\frac{1}{s-m_H^2}\right)-\frac{2kv^2}{\sqrt{s-4m_S^2}\sqrt{s-4m_H^2}}\right.$$
$$\left.\times\ \ln\left(1+\frac{2\sqrt{s-4m_S^2}\sqrt{s-4m_H^2}}{s-\sqrt{s-4m_S^2}\sqrt{s-4m_H^2}-2m_h^2}\right)\right),$$

$$a_0(SH\to SH) = \frac{k}{16\pi}\kappa_{SH\to SH}\left(1+\frac{kv^2}{s-m_H^2}-\frac{3s}{A}m_H^2\ln\frac{s-3m_H^2}{s\,(2m_S^2-m_H^2)-(m_H^2-m_S^2)^2}\right.$$
$$\left.-\frac{s}{A}kv^2\ln\frac{\left(s-3m_S^2\right)s+\left(m_H^2-m_S^2\right)^2}{s\,(2m_H^2-m_S^2)}\right), \tag{10}$$

where

$$A = s^2-2s\left(m_H^2+m_S^2\right)+\left(m_H^2-m_s^2\right)^2,$$
$$\kappa_{SS->HH} = \left(1-\frac{4m_S^2}{s}\right)^{1/4}\left(1-\frac{4m_H^2}{s}\right)^{1/4}$$

and

$$\kappa_{SH->SH} = \sqrt{\left(1-\frac{(m_H+m_S)^2}{s}\right)\left(1-\frac{(m_H-m_S)^2}{s}\right)}$$

are kinematical factors.

Perturbative unitarity will be imposed in the high energy limit $s\gg m_H^2$, m_S^2. The coupled amplitudes are

$$\begin{pmatrix} a_0^{HH\to HH} & a_0^{HH\to SS} & a_0^{HH\to SH} \\ a_0^{SS\to HH} & a_0^{SS\to SS} & a_0^{SS\to SH} \\ a_0^{SH\to HH} & a_0^{SH\to SS} & a_0^{SH\to SH} \end{pmatrix} \underset{s\gg m_H^2,m_S^2}{\longrightarrow} \frac{1}{16\pi}\begin{pmatrix} 3\lambda & k & 0 \\ k & \lambda_S & 0 \\ 0 & 0 & k \end{pmatrix}.$$

Requiring $|\mathrm{Re}\ a_0| \leq \frac{1}{2}$ for each individual process above we obtain

$$HH \to HH \quad m_H \leq \sqrt{\frac{8\pi}{3}}v\,, \tag{11}$$
$$HS\to HS \text{ and } HH \to SS \quad |k| \leq 8\pi\,, \tag{12}$$
$$SS \to SS \quad \lambda_S \leq 8\pi\,. \tag{13}$$

While (11) is the well known bound in [4], (12) shows that the maximum contribution of the Higgs mechanism to m_S is 900 GeV.

Eq. (12) can be improved for $k<0$ by using (11), (13) in the positivity relation

$$-k \leq \frac{8\pi}{3} \sim 4.2 \ \text{ and } \ \frac{1}{2}kv^2 \gtrsim -(520\ \text{GeV})^2. \tag{14}$$

For $m_H = 150\,\text{GeV}$, $\lambda = 0.38$ this goes into $\frac{1}{2}kv^2 \gtrsim -(230\,\text{GeV})^2$.

These bounds can be refined considering the partial wave unitarity for the three coupled channel system HH, SS, SH and constraining the eigenchannel with the highest eigenvalue. Actually SH decouples from HH and SS and the remaining two eigenvalues are

$$\mu_{1,2} = \frac{3\lambda + \lambda_S}{2} \pm \frac{1}{2}\sqrt{(3\lambda + \lambda_S)^2 + 4k^2}\,,$$

providing the constraint

$$\lambda_S + \frac{k^2}{8\pi} \leq 8\pi - 3\lambda\,. \tag{15}$$

The constraint (15) contains all the previous bounds (11) – (13).

After the new measurement of the top mass [9] the upper bound of the Higgs mass from radiative corrections is 251 GeV at 95 % C.L. and the direct lower bound is 114.5 GeV from LEP2 implying the range 0.21–0.97 for λ. We see, however, that for $\lambda = 0.21$–0.97 ($m_H = 114.5\,\mathrm{GeV}$–$251\,\mathrm{GeV}$) the right hand side of (15) changes very small, 24.5–22.1, and $\lambda_S + k^2/(8\pi) \lesssim 8\pi$. Only a heavy Higgs would provide a stronger upper bound.

In conclusion we have considered a simple non-baryonic dark matter candidate model added to the Standard Model. We have calculated the $J = 0$ partial wave amplitudes for the two particle elastic scattering processes and found that the scalar couplings of the model are restricted.

In general our results did not restrict the S mass, however, assuming m_S comes from the Higgs mechanism we get $m_S < 900\,\mathrm{GeV}$. The model can account for the dark matter in the Universe and is consistent with the limits from CDMS-II. Perturbative unitarity constraints allow also higher λ_S, k than those obtained from stability and triviality described in [2].

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