Locality diagrams of conformal models

In what follows, we list known locality diagrams associated to conformal models (see 1 and 2 for background), ordered by increasing number of vertices (equilocality classes). The pictures represent the reduced locality diagrams, i.e. with the universal vertex (corresponding to the equilocality class of the vacuum) left out (note that the universal vertex is never essential but always self-adjacent). Vertices with a red boundary are the loopy vertices, i.e. those adjacent to themselves, while those with a green background are the essential ones, i.e. those whose omission would change the associated lattice. Each diagram is preceded by a conventional alphanumeric label (generated by the classification algorithm) written in bold, and a list of conformal models having the corresponding locality diagram, with the following labeling conventions[1] :

X_n^k with $\mathrm{X}\in\{A,B,C,D,E,F,G\}$ denotes the affine WZNW model of level k based on the simple Lie-algebra X_n of rank n;

`Vir`(n,k) with coprime integers n and k, denotes the minimal Virasoro model of central charge $c=1-6\dfrac{(n-k)^2}{nk}$;

`AT`(n) denotes the Ashkin-Teller model with $n+7$ primaries (the $\mathbb{Z}_2$-orbifold of the free boson compactified on a circle of radius $R=\sqrt{2n}$), i.e. the coset $\dfrac{SO(n)_1\times SO(n)_1}{SO(n)_2}$;

`G`(n) denotes the Gaussian model with $2n$ primaries (the free boson compactified on a circle of radius $R=\sqrt{2n}$), i.e. the $U(1)$ WZNW model at level n;

`SVir`$_1(n)$ denotes the minimal $N=1$ superconformal model of central charge $c=\dfrac{3n(n+6)}{2(n+2)(n+4)}$, i.e. the $\dfrac{SU(2)_n\times SU(2)_2}{SU(2)_{n+2}}$ coset model;

`SVir`$_2(n)$ denotes the minimal $N=2$ superconformal model of central charge $c=\dfrac{3n}{n+2}$, i.e. the $\dfrac{SU(2)_n\times U(1)_4}{U(1)_{2n+4}}$ coset model;

`PF`(n) denotes the parafermionic model of central charge $c=\dfrac{2(n-1)}{n+2}$, i.e. the $\dfrac{SU(2)_n}{U(1)_{n+2}}$ coset;

`D`(G) denotes the holomorphic orbifold with twist group G (trivial cocycle), with $\mathbb{Z}_n,\mathbb{D}_n,\mathbb{S}_n$ and $\mathbb{A}_n$ denoting respectively the cyclic, dihedral, symmetric and alternating groups of degree n;

`T`, `O` and `I` denote the isolated $c=1$ models, i.e. the orbifolds of $SU(2)_1$ with tetrahedral, octahedral and icosahedral respective twist groups.

[1] When not stated explicitly otherwise, in what follows n and k denote arbitrary positive integers, while p,q and r stand for odd primes.

The following table summarizes some of the characteristics of locality diagrams, ordered by increasing number of vertices. The first column gives the conventional label of the diagram, the second the total number $\boldsymbol{\Upsilon}$ of vertices, the third the number $\boldsymbol{\epsilon}$ of essential ones, the fourth the size $|\mathtt{Aut}|$ of the automorphism group (when known), the fifth the number $\boldsymbol{\rho}$ of loops (self-adjacent vertices), the sixth the size $\boldsymbol{\lambda}$ of the associated lattice, the seventh the dimension $\boldsymbol{\delta}$ (length of a maximal chain) of the latter, and finally, the eighth lists some conformal models with the given locality diagram.

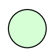
	Υ	ϵ	$\lvert\mathtt{Aut}\rvert$	ρ	λ	δ	examples
58A	2	1	1	1	2	1	A_1^1, A_2^1, A_4^1, A_6^1, E_6^1, E_7^1, E_8^{n+2} $(n<5)$, $F_4^n(n<9)$, G_2^n $(n<9)$, $\mathtt{G}(1)$
1A	3	2	1	2	3	2	$\mathbf{D}(\mathbb{M}_{11})$, $\mathtt{PF}(2)$, A_1^2, A_3^1, A_8^1, B_n^1, C_{2n}^1, D_{2n+3}^1, E_8^2, $\mathtt{G}(2)$, $\mathtt{Vir}(4,3)$
59A	4	2	2	1	4	2	$\mathtt{PF}(p)$, A_1^{2n+1}, $A_2^{3n\pm1}$
9A	4	3	1	2	4	3	$\mathtt{AT}(1)$, A_1^{2n}, A_2^3
62A	4	3	6	4	5	2	$\mathbf{D}(\mathbb{Z}_2)$, D_{4n}^1 $(n\leq 3)$
62B	4	3	2	2	5	2	D_{4n+2}^1 $(n\leq 3)$
84A	5	4	1	3	5	4	$\mathbf{D}(\mathbb{S}_5)$, $\mathbf{D}(\mathbb{S}_6)$, $\mathbf{D}(\mathbb{M}_{10})$, B_4^2
76A	5	4	4	3	6	2	$\mathbf{D}(\mathbb{Z}_3)$
2A	6	3	1	2	6	3	$\mathtt{PF}\left(p^2\right)$, A_3^{2n+1}, A_8^2, A_8^4,D_5^3,D_7^3,D_9^3, E_7^2, $\mathtt{SVir}_2(1)$, $\mathtt{Vir}(5,4)\cong\mathtt{SVir}_1(3)$
11A	6	4	2	2	6	4	$\mathtt{PF}\left(2^2\right)$,$\mathtt{AT}(p)$, A_3^2, A_8^3, D_5^2, D_7^2, D_{11}^2, D_{13}^2
61A	6	5	1	3	6	5	A_3^4, D_5^4, $\mathtt{G}(16)$, $\mathtt{T}$
82A	7	6	4	4	8	4	$\mathbf{D}(\mathbb{S}_3)$
93A	7	6	16	3	8	2	$\mathbf{D}(\mathbb{Z}_5)$
172A	7	6		4	7	6	$\mathtt{G}(32)$
71A	8	3	6	1	8	3	$\mathtt{PF}(15)$, A_5^5, A_5^7, $\mathtt{G}(pq)$
63A	8	4	6	4	10	3	D_4^3, D_4^5, D_8^3
63B	8	4	2	2	10	3	D_6^3, D_{10}^3
3A	8	4	1	2	8	4	A_5^2, A_5^3, A_5^4, A_5^8, A_7^5, $\mathtt{Vir}(n+5,n+4)$, $\mathtt{PF}(2p)$, $\mathtt{G}(4p)$, $\mathtt{G}\left(p^3\right)$, $\mathtt{PF}(27)$
90A	8	5	2	4	8	5	A_5^6
15A	8	5	1	3	8	5	$\mathtt{PF}(8)$,$\mathtt{AT}\left(p^2\right)$, A_7^2, D_9^2
118A	8	7	12	5	9	4	$\mathbf{D}(\mathbb{A}_4)$
91A	8	6	2	3	8	6	A_7^4
12A	8	7	6	4	10	5	$\mathtt{AT}(4)$, D_4^2, D_4^4, D_4^6, D_6^4, D_8^4
174A	8	7		4	8	7	$\mathtt{G}(64)$
83A	9	7	6	6	9	6	$\mathbf{D}(\mathbb{S}_4)$
10A	9	8	4	4	13	4	$\mathtt{AT}(2)$
95A	9	8	96	3	10	2	$\mathbf{D}(\mathbb{Z}_7)$
14A	9	8	6	5	11	6	$\mathtt{AT}(8)$, D_8^2
170A	9	4		4	9	4	$\mathtt{G}\left(2p^2\right)$
180A	9	8		5	9	8	$\mathtt{G}(128)$
17A	10	5	6	2	10	5	$\mathtt{AT}(pq)$
21A	10	6	1	3	10	6	$\mathtt{AT}(27)$, $\mathtt{PF}(16)$
13A	10	7	2	4	12	5	$\mathtt{AT}(2p)$, D_6^2, D_{10}^2, $\mathtt{SVir}_1(4)$

	$\boldsymbol{\Upsilon}$	$\boldsymbol{\epsilon}$	$\lvert\mathtt{Aut}\rvert$	$\boldsymbol{\rho}$	$\boldsymbol{\lambda}$	$\boldsymbol{\delta}$	examples
92A	10	9	16	8	15	4	$\mathbf{D}(\mathbb{Z}_4)$
18A	10	9	6	5	12	7	$\mathtt{AT}(16)$
171A	10	5		3	10	5	$\mathtt{G}(8p)$
23A	11	10	6	6	13	8	$\mathtt{AT}(32)$
5A	12	4	2	2	12	4	$\mathtt{G}\big(p^2q\big)$, $\mathtt{G}(2pq)$, $\mathtt{SVir}_2(p{-}2)$ for $p>3$, $\mathtt{SVir}_1(2n{+}1)$
70A	12	5	2	2	12	5	$\mathtt{PF}(12)$
72A	12	5	1	4	12	5	$\mathtt{PF}(18)$, $\mathtt{G}\big(4p^2\big)$, $\mathtt{G}\big(2p^3\big)$
31A	12	7	1	4	12	7	$\mathtt{AT}(81)$, $\mathtt{PF}(32)$
16A	12	8	2	4	14	6	$\mathtt{AT}(4p)$, D_{12}^2
29A	12	11	6	6	14	9	$\mathtt{AT}(64)$
4A	12	11	2	4	18	5	$\mathtt{SVir}_2(2)$
173A	12	6		3	12	6	$\mathtt{G}(16p)$
19A	13	8	2	6	15	6	$\mathtt{AT}\big(2p^2\big)$
99A	13	12	7680	3	14	2	$\mathbf{D}(\mathbb{Z}_{11})$
165A	13	12		7	15	10	$\mathtt{AT}(128)$
25A	14	6	2	3	14	6	$\mathtt{AT}\big(p^2q\big)$
121A	14	8	2	4	14	6	$\mathtt{AT}\big(3^5\big)$, $\mathtt{PF}\big(2^6\big)$
169A	14	8	2	4	16	6	$\mathtt{SVir}_1(2n{+}2)$
20A	14	9	2	5	16	7	$\mathtt{AT}(8p)$
89A	14	12	144	9	16	8	$\mathbf{D}(\mathbb{M}_9)$
176A	14	7		4	14	7	$\mathtt{G}(32p)$
101A	15	14	-	3	16	2	$\mathbf{D}(\mathbb{Z}_{13})$
108A	15	14	-	4	16	4	$\mathbf{D}(\mathbb{D}_{22})$
175A	15	6		6	15	6	$\mathtt{G}\big(8p^2\big)$, $\mathtt{G}\big(2p^4\big)$
77A	16	15		16	67	4	$\mathbf{D}(\mathbb{D}_4)$
124A	16	5		2	16	5	$\mathtt{PF}(30)$, $\mathtt{G}(4pq)$, $\mathtt{G}(135)$
73A	16	6		3	16	6	$\mathtt{PF}(24)$
22A	16	8		4	18	6	$\mathtt{AT}(2pq)$
24A	16	9		6	18	7	$\mathtt{AT}\big(4p^2\big)$
26A	16	10		5	18	8	$\mathtt{AT}(16p)$
27A	16	9		6	18	7	$\mathtt{AT}(54)$
122A	16	9		5	16	9	$\mathtt{AT}\big(3^6\big)$
177A	16	4		1	16	4	$\mathtt{G}(105)$ $(=\mathtt{G}(pqr)?)$
178A	16	6		4	16	6	$\mathtt{G}(108)$ $(=\mathtt{G}\big(4p^3\big)?)$
162A	18	6		2	18	6	$\mathtt{AT}(105)$
65A	32	13	2	4	44	7	$\mathtt{SVir}_2(4p{-}2)$

Diagrams

2 vertices

58A: A_1^1, A_2^1, A_4^1, A_6^1, E_6^1, E_7^1, E_8^{2+n}, F_4^n, G_2^n

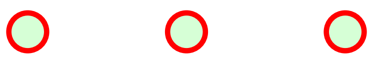


3 vertices

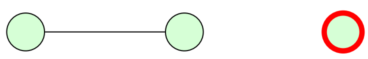
1A: $\mathtt{Vir}(4,3)$, A_1^2, A_3^1, A_8^1, B_n^1, C_2^1, $D_{\mathtt{odd}}^1$, E_8^2

4 vertices

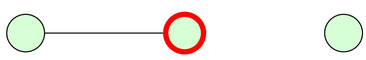
62A: $\mathsf{D}(\mathbb{Z}_2)$, D_4^1, D_8^1, D_{12}^1



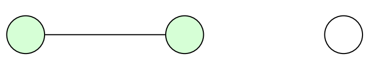
62B: D_6^1, D_{10}^1, D_{14}^1



9A: $\mathtt{AT}_1$, A_1^{2n}, A_2^{3n}, A_4^{5n}, A_6^7, A_7^1, $B_n^{>1}$, $C_{\mathtt{even}}^k$, $C_{\mathtt{odd}}^{\mathtt{even}}$, E_6^{3n}, E_7^4



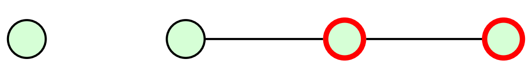
59A: $A_1^{\mathtt{odd}}$, $A_2^{3n\pm1}$, A_4^2, $A_4^{5n\pm1}$, $A_4^{5n\pm2}$, A_5^1, $A_6^{2,3}$, $C_n^{\mathtt{odd}}$, D_5^1, $\mathtt{PF}(p)$



5 vertices

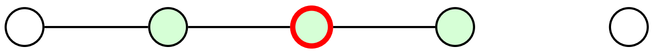
76A: $\mathsf{D}(\mathbb{Z}_3)$

84A: B_4^2, $\mathsf{D}(\mathbb{S}_5)$, $\mathsf{D}(\mathbb{S}_6)$, $\mathsf{D}(M_{10})$

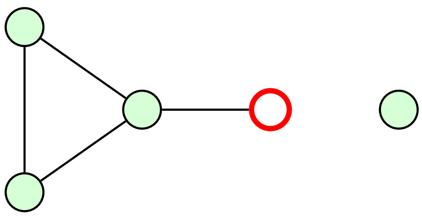


6 vertices

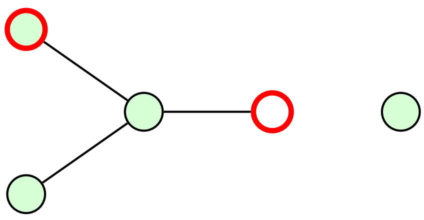
2A: $\mathtt{Vir}(5,4)$, $\mathtt{SVir}_2(1)$, A_3^3, D^3_{3+2n}, $\mathtt{PF}(p^2)$



11A: $\mathtt{AT}(p)$, A_3^{2+4n},D_5^{2+4n}, $\mathtt{PF}(4)$

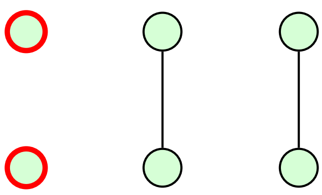


61A: A_3^{4n}, $\mathbf{T}$

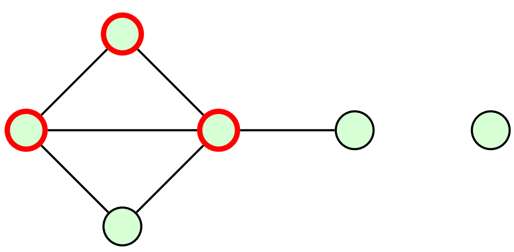


7 vertices

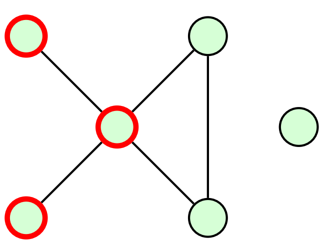
93A: $\mathsf{D}(\mathbb{Z}_5)$



172A: $\mathtt{G}(32)$

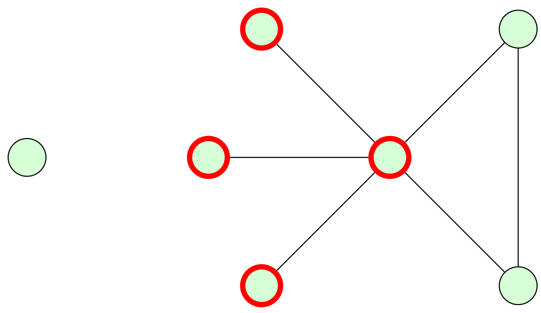


82A: $\mathsf{D}(\mathbb{S}_3)$

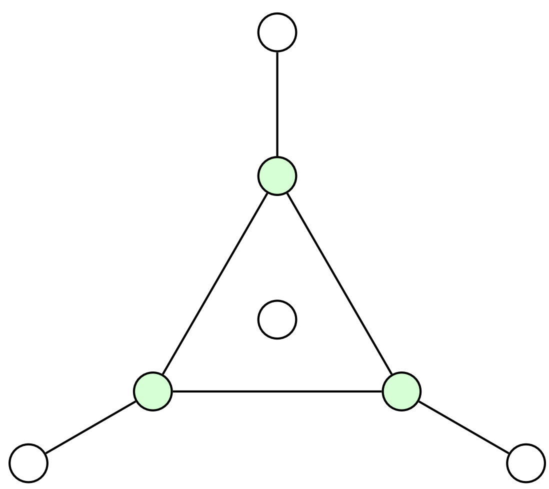


8 vertices

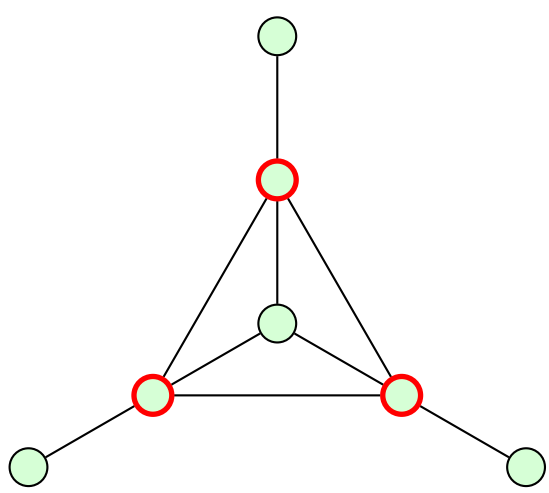
118A: $\mathbf{D}(\mathbb{A}_4)$



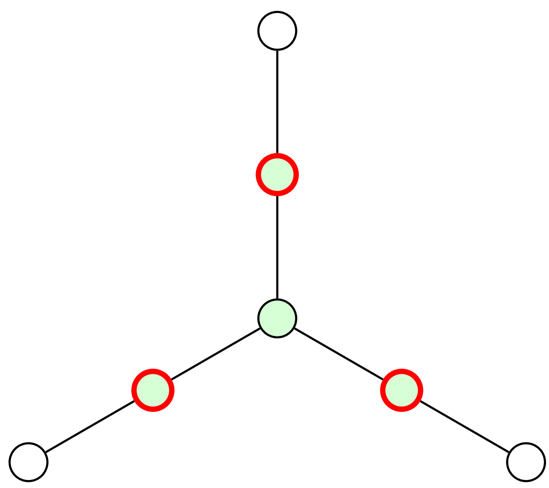
71A: A_5^5, A_5^7, $\texttt{PF}(15)$



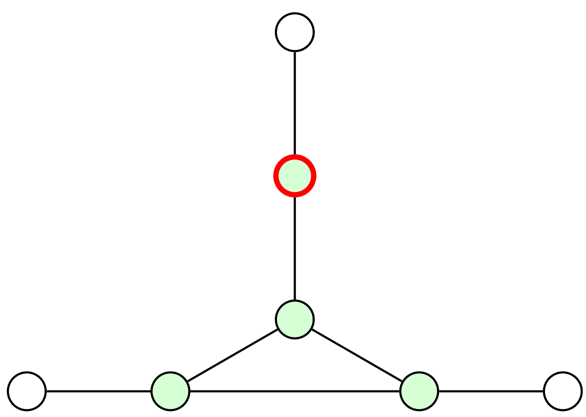
12A: $D_{4n}^{\mathbf{even}}$



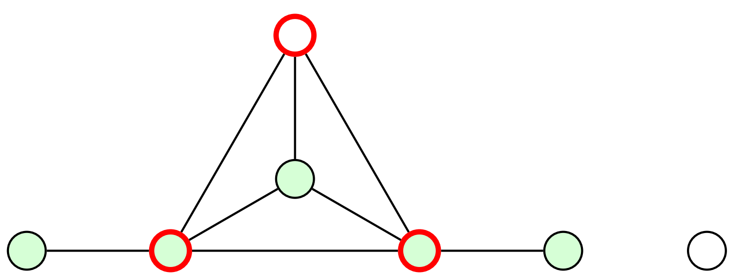
63A: $D_{4n}^{\mathbf{odd}}$



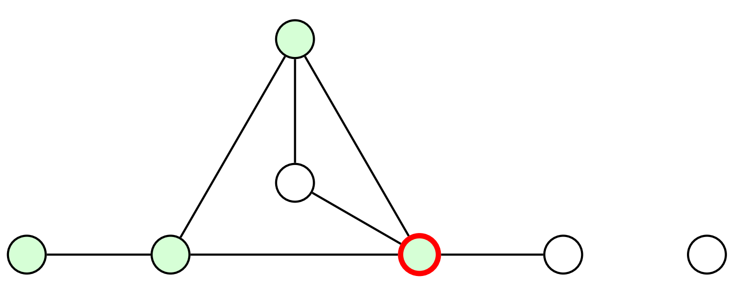
63B: D_6^3, D_{10}^3



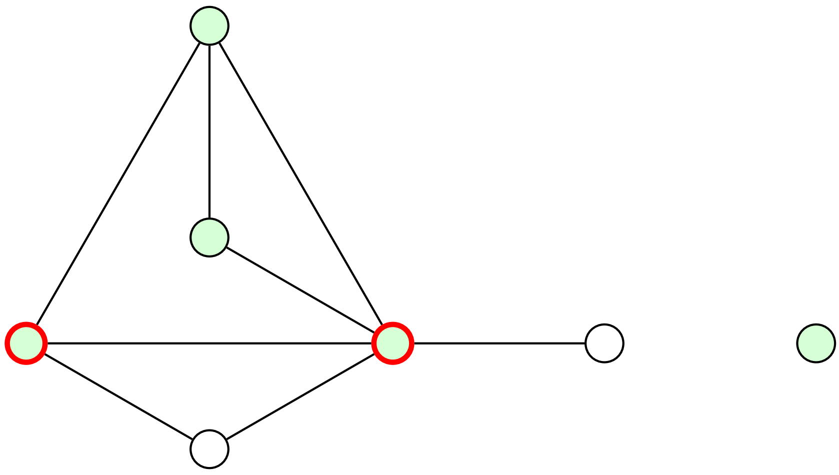
90A: A_5^6



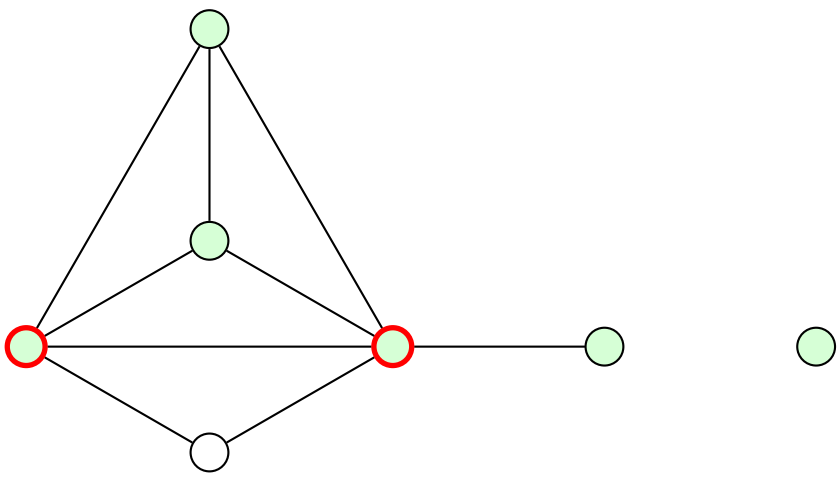
3A: `Vir`, A_5^3, `PF`(27), `PF`$(2p)$



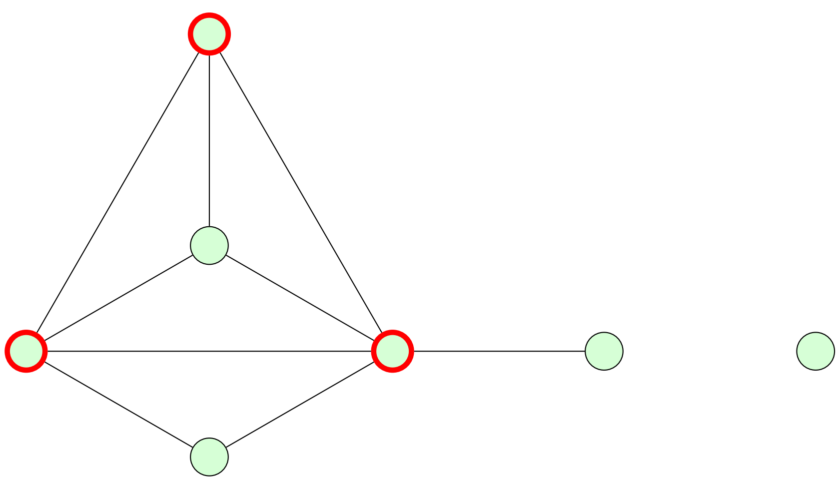
15A: A_7^2, D_9^2, $\mathtt{PF}(8)$, $\mathtt{AT}(p^2)$



91A: A_7^4

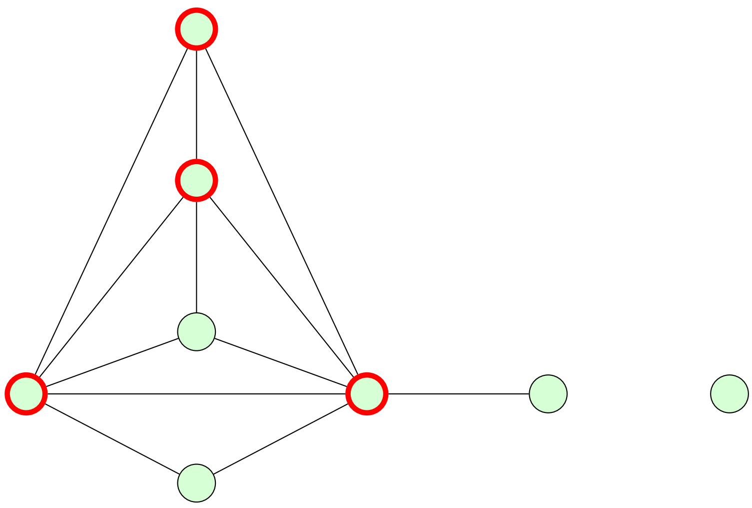


174A: $\mathtt{G}(64)$

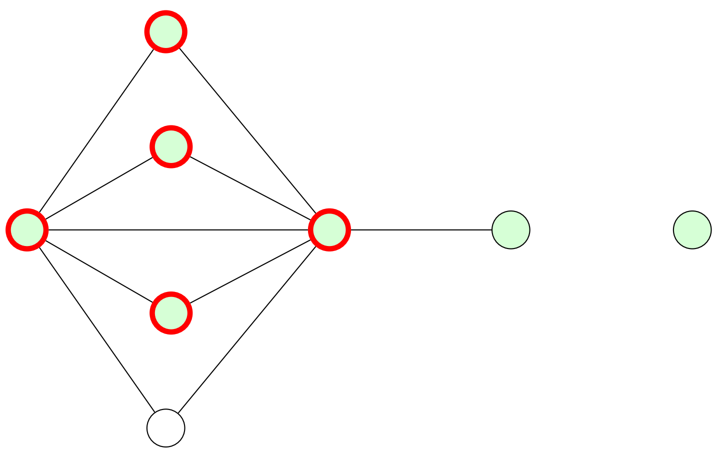


9 vertices

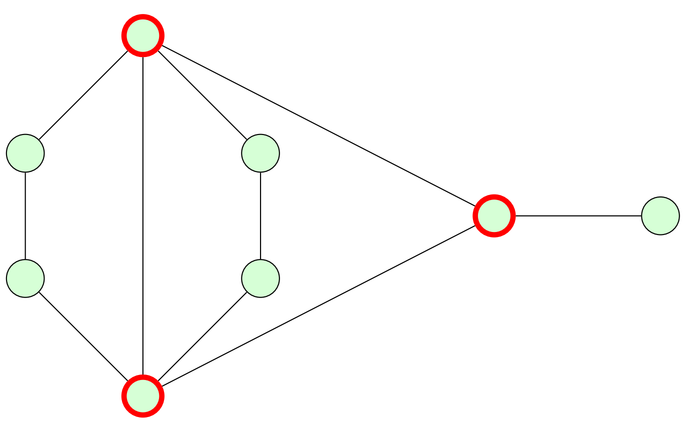
180A: $\mathtt{G}(128)$



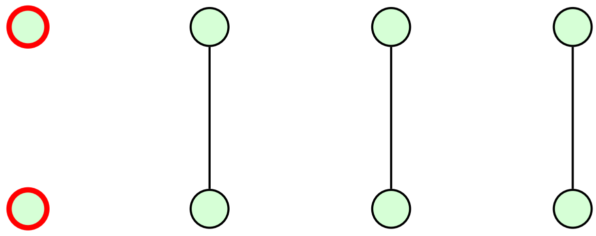
83A: $\mathbf{D}(\mathbb{S}_4)$



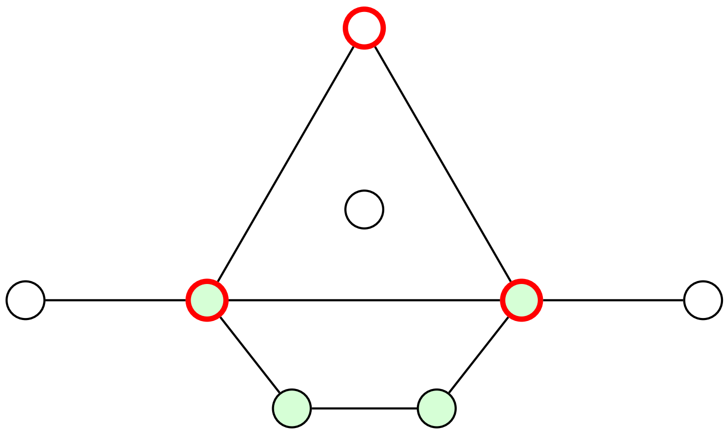
10A: $\mathtt{AT}(2)$



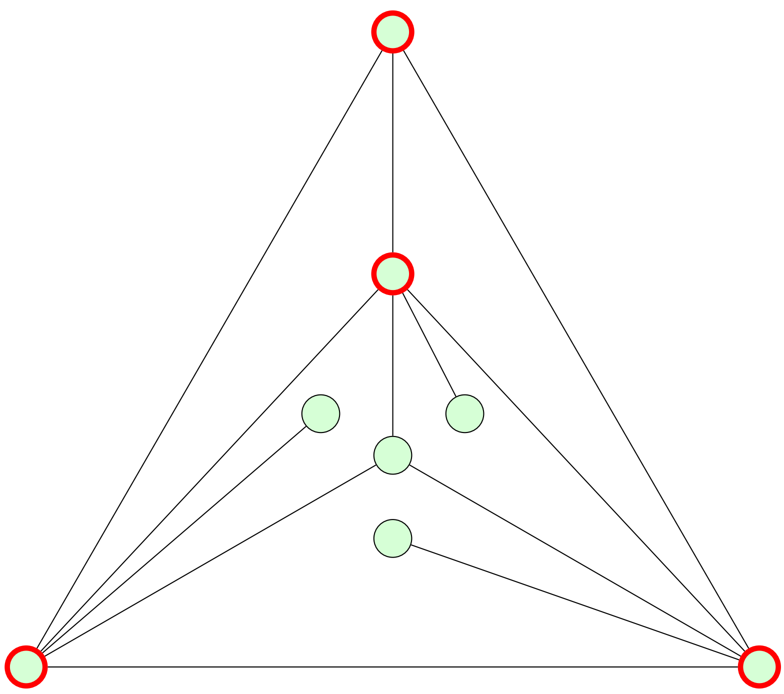
95A: $\mathsf{D}(\mathbb{Z}_7)$



170A: $\mathtt{G}(2p^2)$

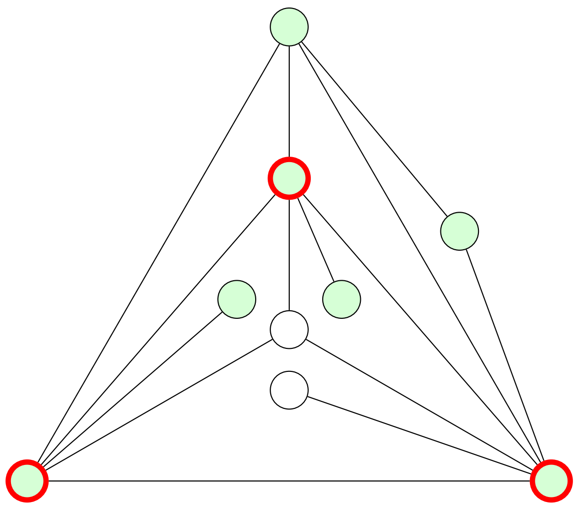


14A: $\mathtt{AT}(8)\,, D_8^2$

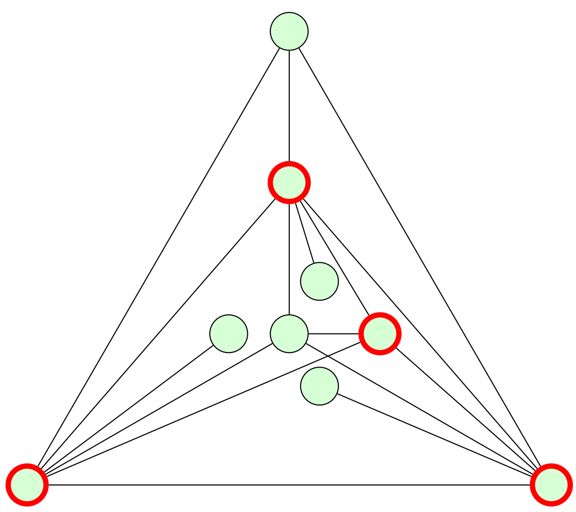


10 vertices

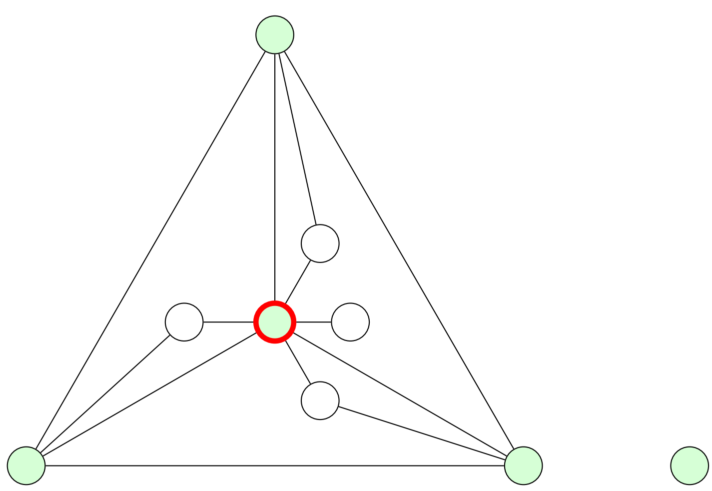
13A: $\texttt{AT}(2p)$, D_6^2, D_{10}^2, $\texttt{SVir}_1(4)$



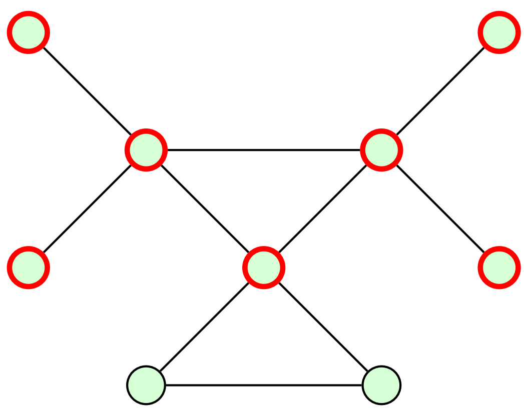
18A: $\texttt{AT}(16)$



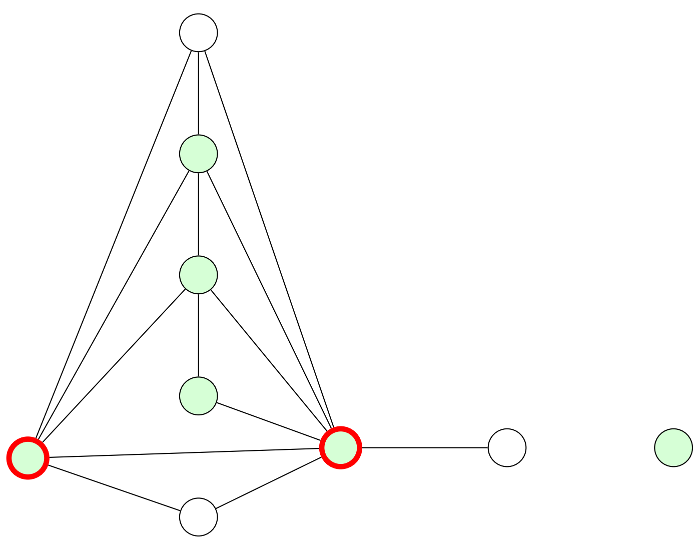
17A: $\texttt{AT}(pq)$



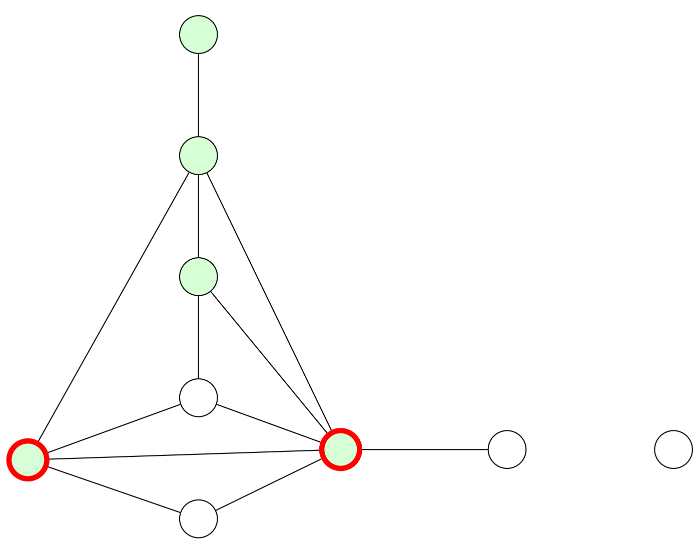
92A: $\mathbf{D}(\mathbb{Z}_4)$



21A: $\texttt{PF}(16)$, $\texttt{AT}(p^3)$

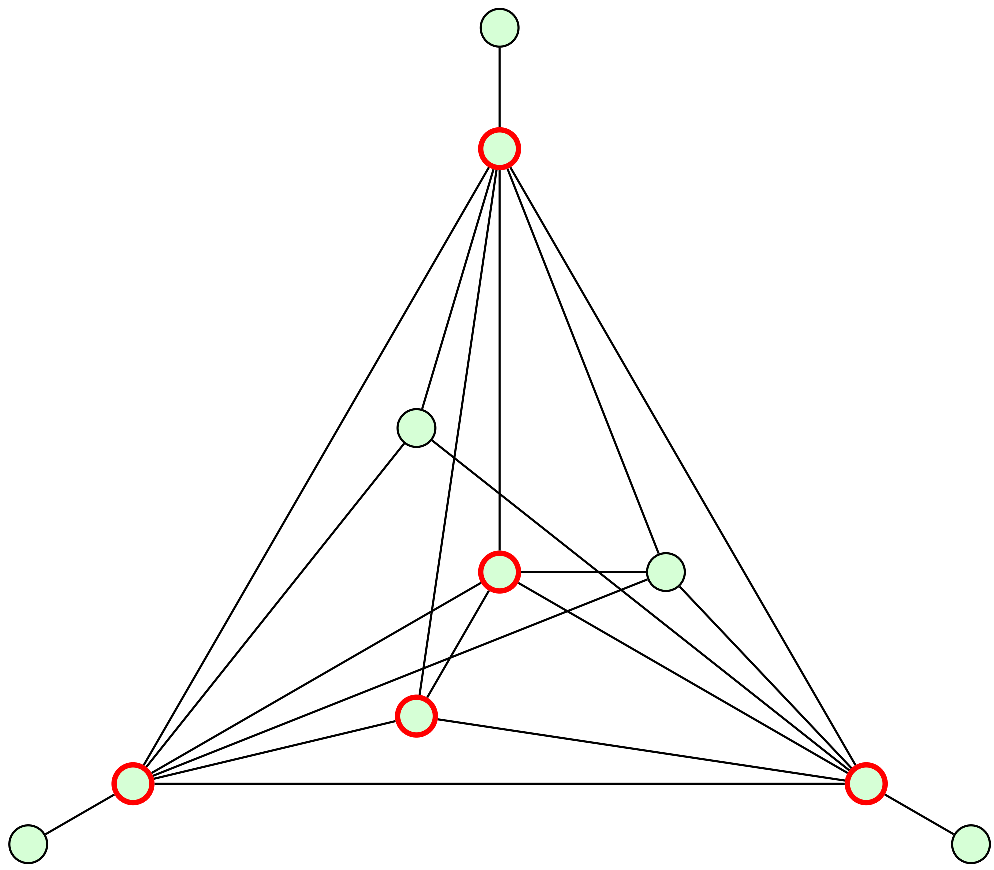


171A: $\texttt{G}(8p)$



11 vertices

23A: $\mathtt{AT}(32)$



12 vertices

5A: $\mathtt{G}(p^2q)\,,\mathtt{G}(2pq)\,,\mathtt{SVir}_1(2n+3)\,,\mathtt{SVir}_2(p{-}2)$ for $p>3$