ISSUES ON MAGNON REFLECTION

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- explaining poles of magnon reflection amplitudes
  by Landau type diagrams

- extension of boundary state formalism
  for magnon reflections

Introduction

integrability of planar $\mathcal{N} = 4$ SYM $\leftrightarrow$ AdS/CFT correspondence

planar $\mathcal{N} = 4$ SYM $\leftrightarrow$ 2d theory world sheet of string

single trace operators: closed spin chain dilatation operator

states with large charges: long closed string on a circle

infinite string limit : dispersion relation, scattering amplitudes of fundamental excitations ($=$ magnons)

non relativistic integrable model

open spin chains in $\mathcal{N} = 4$ SYM open strings with D branes integrability preserving boundaries

(Hofman Maldacena) giant graviton with two orientations relative to open string ground state

W, Y, Z scalar fields in $\mathcal{N} = 4$ SYM

$Y = 0$ brane

$$\mathcal{O}_Y = \epsilon^{j_1 \dots j_{N-1} A}_{i_1 \dots i_{N-1} B} Y^{i_1}_{j_1} \dots Y^{i_{N-1}}_{j_{N-1}} (Z \dots Z \chi Z \dots Z \chi' Z \dots)^B_A$$

$Z = 0$ brane

$$\mathcal{O}_Z = \epsilon^{j_1 \dots j_{N-1} A}_{i_1 \dots i_{N-1} B} Z^{i_1}_{j_1} \dots Z^{i_{N-1}}_{j_{N-1}} (\chi Z \dots Z \chi Z \dots ' Z \chi'' Z \dots Z \chi''')^B_A$$

- spin chain connected to giant graviton through boundary impurities χ, χ'''
- there are boundary degrees of freedom
- full symmetry of the bulk preserved

(Hofman Maldacena)	matrix structure of magnon reflection amplitudes by residual symmetries
	Boundary Crossing Condition (BCC) for $Y = 0$
	boundary bound states for $Z = 0$
(Chen Correa)	solution of BCC for $Y = 0$
	BCC for $Z = 0$
(Ahn Bak Rey)	solution of BCC for $Z = 0$
	multimagnon reflections
(Ahn Nepomechie)	ZF algebra for both branes

Magnon kinematics, spectrum, S matrix

magnon 16 dim. (short) BPS repr. of $SU(2|2) \times SU(2|2)$ SUSY

$$E = \Delta - J = \sqrt{1 + 16 g^2 \sin^2\left(\frac{p}{2}\right)} \quad \text{where} \quad g = \sqrt{g_{YM}^2 N}/4\pi$$

$x^{\pm}$ spectral parameters

$$p = p(x^{\pm}) = \frac{1}{i} \log\left(\frac{x^+}{x^-}\right) \qquad E = E(x^{\pm}) = \frac{g}{i}\left[\left(x^+ - \frac{1}{x^+}\right) - \left(x^- - \frac{1}{x^-}\right)\right]$$

constraint $\left(x^+ + \frac{1}{x^+}\right) - \left(x^- + \frac{1}{x^-}\right) = \frac{i}{g}$

$Q \in \mathbb{N}$ magnon bound states $E = \sqrt{Q^2 + 16 g^2 \sin^2\left(\frac{P}{2}\right)}$

$$x_j^- = x_{j+1}^+ \quad \text{for} \quad j = 1, \ldots, Q-1 \qquad X^+ \equiv x_1^+ \quad X^- \equiv x_Q^-$$

velocity $v(X^{\pm}) = \frac{dx}{dt} = \frac{1}{2g}\frac{dE}{dP} = \frac{X^+ + X^-}{1 + X^+ X^-}$

magnon-magnon scattering $S(x_1, x_2)$
(Beisert) intertwining property up to phase

$$S_{\text{full}} = S_0^2(x_1, x_2)\Big(\widehat{S}_{su(2|2)}(x_1, x_2) \otimes \widehat{S}_{su'(2|2)}(x_1, x_2)\Big)$$ “string” basis

$$S_0^2(x_1, x_2) = \frac{(x_1^- - x_2^+)}{(x_1^+ - x_2^-)} \frac{(1 - \frac{1}{x_1^+ x_2^-})}{(1 - \frac{1}{x_1^- x_2^+})} \sigma^2(x_1^\pm, x_2^\pm) \equiv \frac{\tilde{S}(x_1^\pm, x_2^\pm)}{A(x_1^\pm, x_2^\pm)}$$

$A(x_1^\pm, x_2^\pm) = \frac{(x_1^+ - x_2^-)}{(x_1^- - x_2^+)}$ $\sigma(x_1^\pm, x_2^\pm)$ dressing phase

$\phi(x_1)$ magnon’s hw $|\phi(x_1)\phi(x_2)\rangle = A(x_1^\pm, x_2^\pm)\tilde{S}(x_1^\pm, x_2^\pm)|\phi(x_2)\phi(x_1)\rangle$

$A(x_1^\pm, x_2^\pm)\tilde{S}(x_1^\pm, x_2^\pm) = \sigma^2(x_1^\pm, x_2^\pm)S_{\text{BDS}}(x_1^\pm, x_2^\pm)$ BDS piece of S matrix factorization $\Psi_{Q_1}(X_1)\ \Psi_{Q_2}(X_2)$ scattering by fusion

$$S_{\text{full}}|\Psi_{Q_1}(X_1)\Psi_{Q_2}(X_2)\rangle =$$

$$A(X_1^\pm, X_2^\pm)\tilde{S}(X_1^\pm, X_2^\pm) \prod_{k=0}^{Q_2-1} F(X_1^\pm, X_2^\pm, k)|\Psi_{Q_2}(X_2)\Psi_{Q_1}(X_1)\rangle$$

$$F(X_1^\pm, X_2^\pm, k) = \left(\frac{X_1^+ + \frac{1}{X_1^+} - X_2^+ - \frac{1}{X_2^+} + \frac{ik}{g}}{X_1^- + \frac{1}{X_1^-} - X_2^- - \frac{1}{X_2^-} - \frac{ik}{g}} \right)^{2-\delta_{k,0}}$$

1st and 2nd order poles

(Dorey Hofman Maldacena; Dorey Okamura)

physical poles $\leftrightarrow$ explanation Landau type diagrams

“physicality condition” close to real axis in any of

- $g \to \infty$ P fixed giant magnon limit
- $g \to \infty$ $k \equiv 2gP$ fixed plane wave limit
- $g << 1$ Heisenberg spin chain limit

reflection on the $Y = 0$ brane

$\phi(x)$ magnon's hw   singlet under $su(1|2) \otimes su(1|2)$

$$R^{Y}_{R\,\text{full}}|\phi(x^{\pm})\rangle = -\sigma(x^{\pm}, -x^{\mp})|\phi(-x^{\mp})\rangle \qquad R^{Y}_{L\,\text{full}} : x^{\pm} \to -x^{\mp}$$

magnon bound state $\Psi_Q(X)$ fusion procedure   (Ahn Bak Rey)

$$R^{Y}_{R\,\text{full}}|\Psi_Q(X^{\pm})\rangle = -\sigma^{-1}(-X^{\mp}, X^{\pm}) \prod_{k=1}^{Q-1} \left(\frac{X^{+} + \frac{1}{X^{+}} - \frac{ik}{2g}}{X^{-} + \frac{1}{X^{-}} + \frac{ik}{2g}} \right) |\Psi_Q(-X^{\mp})\rangle$$

poles   at $X^{-} + \frac{1}{X^{-}} = i\frac{m}{2g}$ $m = -(Q-1), \ldots, -1, 1, 2, \ldots$

zeroes   at $X^{+} + \frac{1}{X^{+}} = i\frac{l}{2g}$ $l = (Q-1), \ldots, 1, -1, -2, \ldots$

no bound states   Bethe-Yang eq. not consistent

poles are physical   for $g \to \infty$ $E \to 4g$

$$P = \pi - iq \qquad q = \ln \frac{2Q + m + \sqrt{16g^2 + (2Q+m)^2}}{\sqrt{16g^2 + m^2} - m} \sim \frac{Q+m}{2g}$$

boundary Coleman Thun mechanism for magnons

(Coleman Thun) poles of exact S matrices $\leftrightarrow$ anomalous thresholds
sometimes even first order poles

(Dorey Tateo Watts; Mattsson Dorey; Bajnok P Takacs) BIQFT

warning naive degree $N-2L$ higher cancellation, zeroes

CT mechanism Landau eq.-s singularities of Feynman diagrams

magnon theory non relativistic $\Pi(E,P)=\frac{i}{E^2-16g^2\sin^2\frac{P}{2}-Q^2+i\epsilon}$

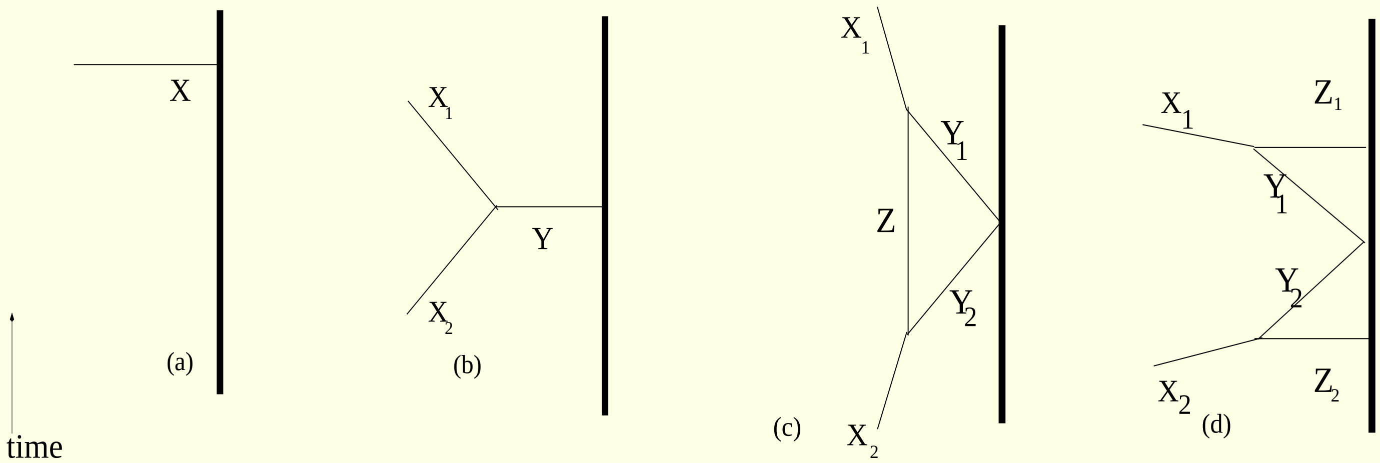
$$E_j^2-16g^2\sin^2\frac{P_j}{2}-Q_j^2=0 \qquad j=1,\dots I \qquad \text{for internal lines}$$

$$\sum_{i\in L_l}\alpha_i E_i=0 \qquad -8g^2\sum_{i\in L_l}\alpha_i\sin P_i=0 \qquad l=1,\dots L \quad \text{for loops}$$

space time diagrams $x_1^0-x_2^0=\alpha E$ $x_1^1-x_2^1=\alpha\sin P$

boundary: only E conserved $X^\pm\to -X^\mp$

Landau diagrams for reflections on the $Y = 0$ brane



at bulk vertices energy, momentum, $U(1)$ charge conserved   vertex conditions

$$X^- = Z^- \qquad X^+ = Y^+ \qquad Z^+ = Y^-$$

$$X^+ = Z^+ \qquad X^- = Y^- \qquad Z^- = Y^+$$

$$X^- = Z^- \qquad X^+ = \frac{1}{Y^-} \qquad Z^+ = \frac{1}{Y^+}$$

$$X^+ = Z^+ \qquad X^- = \frac{1}{Y^+} \qquad Z^- = \frac{1}{Y^-}$$

algorithm to check diagrams

1. choose one admissible vertex condition at every bulk vertex
2. impose the reflection condition at the vertex on the boundary
3. check that it results in the reflection of the external legs $X_1^{\mp} = -X_2^{\pm}$
4. check whether Q for the internal lines is physical ($Q \geq 0$)
5. check that at all (bulk) vertices the particles are indeed heading towards/away from the boundary
6. check whether the internal reflection/scattering amplitudes have zeroes or poles to modify the naive counting of the degree of the pole ($N - 2L$)

horizontal lines $E = 0$ $X^- + \frac{1}{X^-} = -\frac{i}{2g}Q,$ $X^+ + \frac{1}{X^+} = \frac{i}{2g}Q$

not in set of poles diagram (a) ruled out

diagram (b) also ruled out:

insisting on $X_1^{\pm} = -X_2^{\mp}$ extra conditions on $X_2^{\pm}$ (e.g. $X_2^+ = -X_2^-$)

triangle diagram (c) vertical line $P = \pm\pi$ $Z^+ = -Z^-$

Landau eq.-s satisfied:

$Y_1^{\pm} = -Y_2^{\mp}$ implies $P_1^Y = -P_2^Y$ $E_1^Y = E_2^Y$ thus

$\alpha_1 = \alpha_2 = \alpha$ $2\alpha E_1^Y + \alpha_3 E^Z = 0$ solution

naive degree $3 - 2 \cdot 1 = 1$

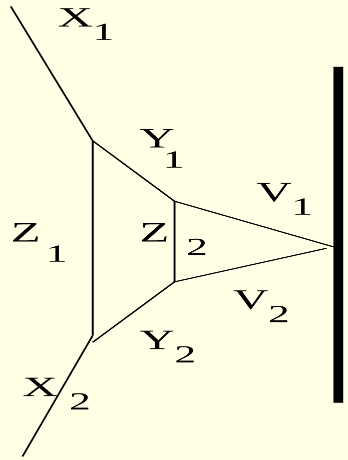
The four triangle diagrams leading to reflection of external legs

vertex conditions	Q values	reflecting magnon
$X_1^- = Z^-, X_1^+ = \frac{1}{Y_1^-}, Z^+ = \frac{1}{Y_1^+}$ $X_2^+ = Z^+, X_2^- = \frac{1}{Y_2^+}, Z^- = \frac{1}{Y_2^-}$	$Q_Z = 2Q + m$ $Q_Y = Q + m$	$Y_2^+ + \frac{1}{Y_2^+} = \frac{im}{2g}$ $Y_2^- + \frac{1}{Y_2^-} = -\frac{i}{2g}(2Q + m)$
$X_1^+ = Z^+, X_1^- = \frac{1}{Y_1^+}, Z^- = \frac{1}{Y_1^-}$ $X_2^- = Z^-, X_2^+ = \frac{1}{Y_2^-}, Z^+ = \frac{1}{Y_2^+}$	$Q_Y = -(Q + m)$	
$X_1^+ = Z^+, X_1^- = Y_1^-, Z^- = Y_1^+$ $X_2^- = Z^-, X_2^+ = Y_2^+, Z^+ = Y_2^-$	$Q_Z = -m$ $Q_Y = Q + m$	$Y_2^+ + \frac{1}{Y_2^+} = \frac{i}{2g}(2Q + m)$ $Y_2^- + \frac{1}{Y_2^-} = -\frac{im}{2g}$
$X_1^- = Z^-, X_1^+ = Y_1^+, Z^+ = Y_1^-$ $X_2^+ = Z^+, X_2^- = Y_2^-, Z^- = Y_2^+$	$Q_Y = -(Q + m)$	

first internal reflection has zero finite

third internal reflection has pole 2^{nd} order pole

velocities: first same sign as $v(X_2)$ third opposite the diagram that works naive degree $6-2\cdot 2=2$



along Z_1: first choice along Z_2: fourth choice

Q values

$$Q_{Z_1}=2Q+m \quad Q_{Y_2}=Q_{Y_1}=Q+m \quad Q_{V_2}=Q_{V_1}=Q \quad Q_{Z_2}=m$$

reflecting magnon $V_2^+ + \frac{1}{V_2^+} = -\frac{im}{2g}$ $V_2^- + \frac{1}{V_2^-} = -\frac{i}{2g}(2Q+m)$

internal reflection has zero 1^{st} order pole indeed !

velocities: diagram consistent

Boundary state formalism for magnon reflections

scalar factors determined from boundary crossing

crossing relations: singlet scatters/reflects trivially

boundary state in BIQFT (Ghoshal Zamolodchikov)

alternative derivation

connection to mirror magnon model (Arutyunov Frolov)

Summary of boundary state formalism in BIQFT

(Ghoshal Zamolodchikov) two *equivalent* Hamiltonian descriptions

(relativistic invariance) open channell

$$A_i^\dagger(\theta)B = R_i^j(\theta)A_j^\dagger(-\theta)B$$

exchanging time and space in Euclidean version (double Wick rotation)

closed channel boundary state $\langle B|$ all information

requirement: correlation functions identical

$$\langle B| = \langle 0|(1 + \int_0^\infty d\theta K^{ab}(\theta)A_a(-\theta)A_b(\theta) + \dots) \qquad \text{out states}$$

$$\langle B| = \langle 0|(1 + \int_0^\infty d\theta K^{ab}(-\theta)A_a(\theta)A_b(-\theta) + \dots) \qquad \text{in states}$$

consistency $K^{dc}(\theta) = K^{ab}(-\theta)S_{ab}^{cd}(2\theta)$ boundary crossing if $K \leftrightarrow R$

reduction formulae $K^{ab}(\theta) = R_a^b(i\frac{\pi}{2} - \theta)$

combined with boundary YB unitarity and crossing of bulk S matrix

$$[K(\theta), K(\theta')] = 0, \qquad \text{where} \qquad K(\theta) = K^{ab}(\theta) A_a(-\theta) A_b(\theta),$$

boundary state $\langle B| = \langle 0| \exp(\frac{1}{2} \int_{-\infty}^{\infty} K(\theta) d\theta)$

Boundary state formalism for magnons

magnon problem: (•) non relativistic
(••) no reduction formulae

(•) mirror magnon model (double Wick rotation) not equivalent

(Arutyunov Frolov) magnon (p, E) mirror magnon ($\tilde{p}$, $\tilde{E}$)

$$p \to 2i\,\text{arcsinh}(\frac{\sqrt{1+\tilde{p}^2}}{4g}) = i\tilde{E} \qquad E = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} \to i\tilde{p}$$

dispersion relation complex torus z generalized rapidity

$p(z) = 2\mathrm{am}(z) \quad \sin\frac{p(z)}{2} = \mathrm{sn}(z,k) \equiv \mathrm{sn}(z) \quad E(z) = \mathrm{dn}(z,k) \equiv \mathrm{dn}(z)$

$$x^{\pm}(z) = \frac{1}{4g}\left(\frac{\mathrm{cn}(z,k)}{\mathrm{sn}(z,k)} \pm i\right)(1+\mathrm{dn}(z,k)) \qquad k^2 = -16g^2 \in \mathbb{R}$$

torus $\quad z \to z \pm 2\omega_1 \quad z \to z \pm 2\omega_2 \quad \omega_1 = 2K(k^2) \quad \omega_2 = 2iK(1-k^2) - 2K(k^2)$

$$g \to \infty \qquad\qquad \omega_1 \to \frac{\log g}{2g} \qquad \omega_2 \to i\frac{\pi}{4g}$$

rescaling $\quad z \to z/(4g) \quad p \to p/(2g)$ keeps $\quad \mathrm{Im}(z)$ finite

converts dispersion relation to $E^2 - p^2 = 1 \qquad z \qquad$ plays the role of θ

torus degenerates into strip $-\pi < \mathrm{Im}(z) < \pi \qquad -\infty < \mathrm{Re}(z) < \infty$

(Arutyunov Frolov) magnon S-matrix analytic continuation $\mathcal{S}(z_1, z_2)$ to entire rapidity torus

z torus for mirror model:

double Wick rotation   shift $z \to \tilde{z} + \frac{\omega_2}{2}$

$$\tilde{p} = -i\mathrm{dn}(\tilde{z} + \frac{\omega_2}{2}, k) = \sqrt{k'}\frac{\mathrm{sn}(\tilde{z})}{\mathrm{cn}(\tilde{z})} \qquad \tilde{E} = 2\mathrm{arccoth}\frac{\sqrt{k'}}{\mathrm{dn}(\tilde{z})}$$

$-\infty < \tilde{p} < \infty$ mapped to $-\omega_1/2 < \tilde{z} < \omega_1/2$

analogous to $\theta \to \theta + i\frac{\pi}{2}$

(Arutyunov Frolov)   mirror magnon S-matrix

$$\tilde{\mathcal{S}}(\tilde{z}_1, \tilde{z}_2) = \mathcal{S}(\tilde{z}_1 + \frac{\omega_2}{2}, \tilde{z}_2 + \frac{\omega_2}{2})$$

boundary state for magnon reflection on $Y = 0$ brane

$$A_i^\dagger(z)B = R_i^j(z)A_j^\dagger(-z)B \qquad i,j = 1\dots 4$$

$$\langle B| = \langle 0|(1 + \int_0^{\omega_1/2} d\tilde{z}\rho(\tilde{z})K^{ab}(\tilde{z})\tilde{A}_a(-\tilde{z})\tilde{A}_b(\tilde{z}) + \dots)$$

consistency: $K^{dc}(\tilde{z}) = K^{ab}(-\tilde{z})\tilde{S}_{ab}^{cd}(\tilde{z},-\tilde{z})$ BCC if $K \leftrightarrow R$

($\bullet\bullet$) K R relation demand $z \to z + \frac{\omega_2}{2}$ continuation of boundary YB combined with $\tilde{S} \leftrightarrow S$

$$\mathcal{S}_{12}(z_1,z_2)\mathcal{S}_{21}(z_2,z_1) = \mathbb{I} \qquad \mathbb{C}^{-1}\mathcal{S}_{12}^{tr}(z_1,z_2)\mathbb{C}\mathcal{S}_{12}(z_1,z_2-\omega_2) = \mathbb{I}$$

should guarantee $[K(z),K(z')] = 0$ $K(z) = K^{ab}(z)\tilde{A}_a(-z)\tilde{A}_b(z)$

$$K^{ab}(z) = \mathbb{C}^{ac}R_c^b(\frac{\omega_2}{2} - z)$$

$\mathbb{C}$ charge conjugation

continuing back $z \rightarrow u + \frac{\omega_2}{2}$

exploiting $R(u)R(-u) = 1$ $S(z_1 + \omega_2, z_2 + \omega_2) = S(z_1, z_2)$

$$\mathbb{C}^{ac_1} R^b_{c_1}(\omega_2 + u) S^{cd}_{ab}(u, -u - \omega_2) R^n_c(u) = \mathbb{C}^{dn}$$

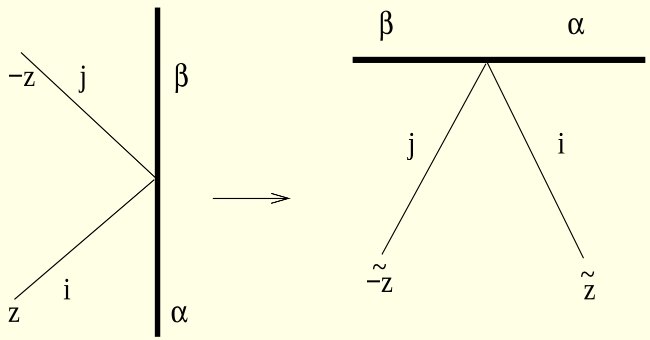
since $x^{\pm}(u + \omega_2) = (x^{\pm}(u))^{-1}$ is the BCC !

boundary state for the $Z = 0$ brane

boundary degree of freedom $B \rightarrow B_\alpha$ $\alpha = 1 \dots 4$

$$A^\dagger_i(z) B_\alpha = R^{j\beta}_{i\alpha}(z) A^\dagger_j(-z) B_\beta$$

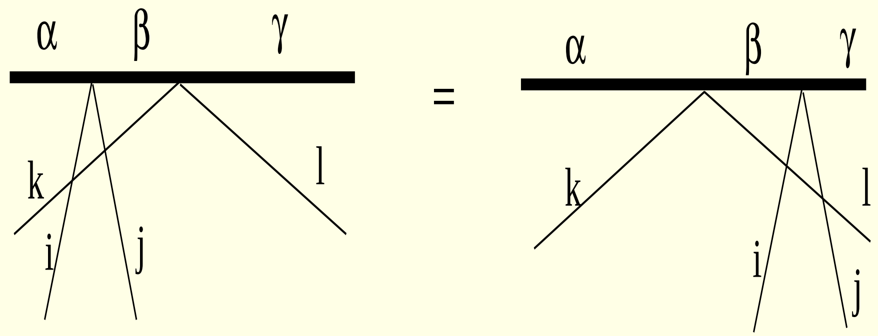
in reflection also boundary changes



boundary state characterized by b.d.o.f $\alpha\ \beta$ $K^{\alpha\beta}(\tilde{z}) = K^{ab\alpha\beta}(\tilde{z})\tilde{A}_a(-\tilde{z})\tilde{A}_b(\tilde{z})$

$$\langle B_{\alpha\beta}| = \langle 0|(\delta_{\alpha\beta} + \int_0^{\omega_1/2} d\tilde{z}\rho(\tilde{z})K^{\alpha\beta}(\tilde{z}) + \int_0^{\omega_1/2} d\tilde{z}\rho(\tilde{z}) \int_0^{\omega_1/2} d\tilde{w}\rho(\tilde{w})K^{\alpha\gamma}(\tilde{z})K^{\gamma\beta}(\tilde{w})$$

consistency: $\boxed{K^{dc\alpha\beta}(\tilde{z}) = K^{ab\alpha\beta}(-\tilde{z})\tilde{S}_{ab}^{cd}(\tilde{z},-\tilde{z})}$ decorated with $\alpha\ \beta$



demanding $[K(\tilde{z}),K(\tilde{w})]^{\alpha\beta} \equiv K^{\alpha\gamma}(\tilde{z})K^{\gamma\beta}(\tilde{w}) - K^{\alpha\gamma}(\tilde{w})K^{\gamma\beta}(\tilde{z}) = 0$

$$K^{ab\alpha\beta}(z) = \mathbb{C}^{ac}R_{c\alpha}^{b\beta}(\frac{\omega_2}{2} - z)$$

continuing back $z \to u + \frac{\omega_2}{2}$ exploiting unitarity of R

consistency gives BCC again but . . . more complicated