Spectral curve for open strings attached to the $Y = 0$ brane

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Plan

- Motivation
- Open monodromy matrix and quasimomenta
- explicit quasimomenta for open strings
- quasiclassical fluctuations of open string solutions
- quasimomenta from the Y system
- Conclusions

Motivation

integrability in AdS/CFT both for closed and open strings

$\updownarrow$

Lax-connection $\rightarrow$ monodromy matrix $\rightarrow$ quasimomenta

8-sheeted Riemann surface algebraic curve (AC)

[Bena Polchinski Roiban] [Beisert Kazakov Sakai Zarembo]

analytic properties $\rightarrow$ AC for classical solutions energy

generalized to small fluctuations / semiclassical corrections

[Gromov Vieira] [Gromov Kazakov Vieira] [Gromov Kazakov Tsuboi]

large volume sol.s of Y system $\rightarrow$ scaling limit $\rightarrow$ AC recovered open strings (gauge th. determinant type operatros)

end on D-branes integrability preserved

monodromy matrix $\rightarrow$ double row monodromy matrix (drmm)

generalize the AC construction

quasimomenta (log of) eigenvalues of drmm

$Y = 0$ brane analysed in details (ABA and Y system / sol.s available)

Open monodromy matrix and quasimomenta

"open" monodromy matrix and classical double row transfer matrix (Mann-Vasquez, Dekel-Oz)

(GS)σM with consistent integrable boundaries

consistent: boundary terms vanish "gluing conditions"

integrable: Lax connection $a_\mu(\lambda, x)$ transport matrix

$$\partial_0 a_1 - \partial_1 a_0 + [a_0, a_1] = 0 \qquad T(\sigma_2, \sigma_1, \lambda) = P \exp\left(\int_{\sigma_1}^{\sigma_2} d\sigma\, a_1(\lambda, \sigma)\right)$$

closed string / periodic b.c. $\Omega_\gamma(\lambda) = T(2\pi, 0, \lambda)$ $\partial_0 \mathrm{Tr}(\Omega_\gamma^n) = 0$

open string / interval of length L integrability preserving b.c.

$$\boxed{\Omega(\lambda) = U_0 T^{-1}(L, 0, -\lambda) U_L T(L, 0, \lambda)} \qquad U_{L,0} \text{ const. matrices } (U^2 = 1)$$

$U_L a_0(\lambda, L) - a_0(-\lambda, L) U_L = 0 \qquad U_0 a_0(\lambda, 0) - a_0(-\lambda, 0) U_0 = 0$

guarantee $\quad \partial_0 \Omega(\lambda) = [a_0(-\lambda, L), \Omega(\lambda)]$ integrability preserving b.c.

spectral curve for open strings replace $\Omega_\gamma(\lambda)$ by $\Omega(\lambda)$

b.c. and $a_\mu(\lambda, x)$ in terms of "currents" (Maurer-Cartan forms) $j_\mu \in \mathcal{G}$

$a_\mu(\lambda, x) = l_1(\lambda) j_\mu(x) + l_2(\lambda) \epsilon_{\mu\nu} j^\nu(x) \qquad l_i(\lambda)$ from ZCC $\equiv$ EOM

$l_i(\lambda)$ rational ($l_1 = \frac{1}{1-\lambda^2} \quad l_2 = \frac{\lambda}{1-\lambda^2}$) $U_{L,0}$ independent of λ

$$[U, j_0]|_B = 0 = \{U, j_1\}|_B$$

split $\mathcal{G} = \mathcal{G}_1 + \mathcal{G}_{-1} \qquad [U, \mathcal{G}_1] = 0 = \{U, \mathcal{G}_{-1}\} \qquad j_0|_B \in \mathcal{G}_1 \quad j_1|_B \in \mathcal{G}_{-1}$

conserved charges: $\tilde{Q} = \int_0^L d\sigma j_0(\sigma) = Q_1 + Q_{-1}$ only Q_1 is conserved

$$\partial_0 \tilde{Q} = \int_0^L d\sigma \partial_0 j_0 = -(j_1(L) - j_1(0))_1 - (j_1(L) - j_1(0))_{-1}$$

$AdS_5 \times S^5$ GSσM (Arutyunov Frolov) Maurer-Cartan one form

$$A = -g^{-1}dg = \sum_{i=0}^{3} A^{(i)} \qquad g \in SU(2,2|4) \qquad A^{(i)} \qquad \mathbb{Z}_4 \ \ \text{component}$$

“moving frame” flat connection L_α “fixed frame” flat connection l_α

$$l_\alpha = gL_\alpha g^{-1} + \partial_\alpha g\, g^{-1} \qquad \sum_{i=0}^{3} gA^{(i)}g^{-1} = \sum_{i=0}^{3} a^{(i)}$$

vanishing curvature $=$ e.o.m. complex variable ζ $\kappa = \pm 1$

$$\mathrm{l}_0 = 1 \qquad \mathrm{l}_1 = \frac{1}{2}(\zeta^2 + \zeta^{-2}) \qquad \mathrm{l}_2 = -\frac{1}{2\kappa}(\zeta^2 - \zeta^{-2}) \qquad \mathrm{l}_3 = \zeta \qquad \mathrm{l}_4 = \zeta^{-1}$$

$$l_\alpha = (\mathrm{l}_1 - 1)a_\alpha^{(2)} + \mathrm{l}_2 \gamma_{\alpha\beta}\epsilon^{\beta\rho}a_\rho^{(2)} + (\mathrm{l}_3 - 1)a_\alpha^{(1)} + (\mathrm{l}_4 - 1)a_\alpha^{(3)}$$

consistent boundaries boundary term vanishes

[DO] ω metric preserving automorhism gluing conditions

$$l(\zeta) = \omega(\bar{l}(\zeta^{-1})) \quad \text{at} \quad \sigma = 0,\ \pi \qquad l^{(i)} = l_\tau^{(i)} + l_\sigma^{(i)} \quad \bar{l}^{(i)} = l_\tau^{(i)} - l_\sigma^{(i)}$$

consistent with κ symmetry breaks half of supersymmetry integrable boundaries transport matrix $T(\sigma_2,\sigma_1,\zeta) = P\,\mathrm{exp}\Big(\int_{\sigma_1}^{\sigma_2} d\sigma l_\sigma(\sigma,\zeta)\Big)$

[DO] define in presence of boundaries

$$\boxed{T_{0,\pi}(\zeta) = U_0 T^{-1}(\pi,0,\zeta^{-1}) U_\pi T(\pi,0,\zeta)} \qquad U_{0,\pi}^2 = \pm 1$$

$\partial_\tau \mathrm{Str}(T_{0.\pi}(\zeta)) = 0$ iff

$$U_\pi l_\tau(\pi,\zeta) U_\pi^{-1} = l_\tau(\pi,\zeta^{-1}) \qquad U_0 l_\tau(0,\zeta^{-1}) U_0^{-1} = l_\tau(0,\zeta)$$

$\omega_U(h) = U\,h\,U^{-1}$ satisfies all reqirements for $Y=0$ brane $U_{0,\pi}$ matrices known $U = i\,\mathrm{diag}(1,-1,1,-1,1,1,-1,-1)$

$T_{0.\pi}(\zeta)$ open monodromy $\mathrm{Str}(T_{0.\pi}(\zeta))$ classical double row transferm

conserved global charges $\zeta = 1-w$ $l_\sigma(1-w) = wJ^\tau/g + \ldots$

e.o.m. $\partial_\alpha J^\alpha = 0$ $\tilde{Q} = \int_0^\pi d\sigma J^\tau$ only $Q \in \tilde{Q}$ $[Q,U]=0$ conserved

$$T_{0.\pi}(\zeta)|_{\zeta=1-w} = U_0 U_\pi \Big(1+\frac{w}{g}(U_\pi^{-1}\tilde{Q}U_\pi + \tilde{Q}) + \ldots\Big) = U_0 U_\pi\Big(1+\frac{2w}{g}Q + \ldots\Big)$$

$$U_\pi^{-1}\tilde{Q}U_\pi = \omega_{U_\pi^{-1}}(\tilde{Q}) \qquad \omega^2_{U_\pi^{-1}} = 1 \qquad g = \frac{\sqrt{\lambda}}{4\pi}$$

symmetry equations for $T_{0.\pi}(\zeta)$

$L_\alpha(\zeta)$ under $\mathbb{Z}_4$ automorphism $\mathcal{K}L_\alpha(\zeta)^{\mathsf{ST}}\mathcal{K}^{-1} = -L_\alpha(i\zeta)$

$$T_{0,\pi}(i\zeta) = \tilde{\mathcal{K}}(T_{0,\pi}^{-1}(\zeta))^{\mathsf{ST}}(\tilde{\mathcal{K}})^{-1} \quad (\bullet) \qquad \tilde{\mathcal{K}} = g(0)^{-1}\mathcal{K}(g(0)^{-1})^{\mathsf{ST}}$$

$$T_{0,\pi}(\zeta^{-1}) = U_0 T_{0,\pi}^{-1}(\zeta) U_0^{-1} \quad (\bullet\bullet)$$

$4+4$ eigenvalues $(\lambda_1, \ldots \lambda_4|\mu_1, \ldots \mu_4)$ of $T_{0,\pi}(\zeta)$

in terms of quasi-momenta of the S^5 and AdS_5

$$\lambda_i = e^{-i\tilde{p}_i(\zeta)} \qquad \mu_i = e^{-i\hat{p}_i(\zeta)} \qquad i = 1, \ldots 4$$

Analytical properties

$T_{0.\pi}(\zeta)$ depends analytically on ζ $(\zeta \neq 0,\ \infty)$ but $\lambda_i\ \mu_i$ don't

define $Y(\zeta)$: $m(\zeta)Y(\zeta)m^{-1}(\zeta) = -i\zeta\frac{d}{d\zeta}\log(m(\zeta)T_{0,\pi}(\zeta)m^{-1}(\zeta))$

eigenvalues by characteristic function

$$F(\tilde{y}(\zeta),\zeta) = 0, \qquad F(\hat{y}(\zeta),\zeta) = \infty, \qquad F(y,\zeta) = \frac{\tilde{P}(\zeta)}{\hat{P}(\zeta)}\mathrm{sdet}\Big(y - Y(\zeta)\Big)$$

symmetry eq.s imply $F(y,i\zeta) = F(-y,\zeta)$ and $F(y,\zeta^{-1}) = F(y,\zeta)$
thus $F(y,\zeta)$ depends on y^2 $y(\zeta^2+\zeta^{-2})$ $\zeta^4+\zeta^{-4}$
y is a function of $\zeta^2+\zeta^{-2}$

we can introduce $\boxed{x = \frac{1+\zeta^2}{1-\zeta^2}}$ same as for closed string

open monodromy matrix function of x $T_{o,\pi}(x)$

$$\zeta \to 1/\zeta \ \sim\ x \to -x \qquad \zeta \to i\zeta \ \sim\ x \to 1/x$$

reflection inversion

Symmetry and analitic properties of quasimomenta

$x \to 1/x$ $\quad (\zeta \to i\zeta)$ $\quad$ inversion symmetry

$$\tilde{p}_{1,2}(x) = -\tilde{p}_{2,1}(1/x) \qquad \tilde{p}_{3,4}(x) = -\tilde{p}_{4,3}(1/x)$$

$$\hat{p}_{1,2}(x) = -\hat{p}_{2,1}(1/x) \qquad \hat{p}_{3,4}(x) = -\hat{p}_{4,3}(1/x)$$

$x \to -x$ $\quad (\zeta \to 1/\zeta)$ $\quad$ reflection symmetry

$$\tilde{p}_i(-x) = -\tilde{p}_i(x) \qquad \hat{p}_i(-x) = -\hat{p}_i(x) \qquad i = 1, \ldots, 4$$

l_α singularities at $x = \pm 1$ $\to$ simple poles for $\tilde{p}_i$ $\hat{p}_i$

Virasoro constraint $\mathrm{str}(l_\alpha^2) = 0$ inversion reflection

$$\{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4 | \hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4\} \sim \frac{x}{x^2-1}\{\alpha, \alpha, \beta, \beta | \alpha, \alpha, \beta, \beta\}$$

$T_{0.\pi}(\zeta)|_{\zeta=1-w}$ implies at $x \to \infty$

$$\mathrm{diag}(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4 | \hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4) \sim \frac{2}{gx} i Q_{\mathrm{diag}}$$

$\{\tilde{p}_i(x)|\hat{p}_i(x)\}$ quasimomenta eight sheeted Riemann surface

single poles at $x = \pm 1$

square root branch cuts connecting two $\tilde{p}_i(x)/\hat{p}_i(x)$ bosonic

single poles simultaneously on $\tilde{p}_i(x)$ $\hat{p}_j(x)$ sheets fermionic

must respect inversion and reflection generic ones in fourfold multiplets

Explicit quasimomenta for circular open strings

idea: cut into "half" a rotating closed string solution [Stefanski]

$Y = 0$ brane b.c. satisfied $(\partial_\sigma X = \partial_\sigma Z = 0)$

$AdS_5 \times S^5$ metric

$$
\begin{aligned}
ds^2 &= -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\alpha^2 + \sin^2\alpha d\Phi^2 + \cos^2\alpha d\phi^2) \\
&+ d\gamma^2 + \cos^2\gamma d\phi_1^2 + \sin^2\gamma(d\psi^2 + \cos^2\psi d\phi_2^2 + \sin^2\psi d\phi_3^2)
\end{aligned}
$$

S^5 coordinates $X = \cos\gamma e^{i\phi_1}$ $Z = \sin\gamma\cos\psi e^{i\phi_2}$ $Y = \sin\gamma\sin\psi e^{i\phi_3}$

$Y = 0$ brane ($\psi \equiv 0$ or $\psi \equiv \pi$) S^3 : $d\gamma^2 + \cos^2\gamma d\phi_1^2 + \sin^2\gamma d\phi_2^2$

the class of solutions $\rho \equiv 0$

$$\gamma \equiv \frac{\pi}{2} \quad \leftrightarrow \quad X \equiv 0 \qquad Z = \cos(n\sigma)e^{iw\tau} \qquad Y = \sin(n\sigma)e^{iw\tau} \qquad t = \kappa\tau$$

$$\sigma \in (0,\pi) \quad n = 2N \qquad w^2 = n^2 + \nu^2 \qquad \kappa^2 = \nu^2 + 2n^2 \quad \nu \quad \text{const.}$$

energy and angular momenta $E = \frac{1}{2}\sqrt{\lambda}\kappa \qquad J_Z = J_Y = \frac{1}{4}\sqrt{\lambda}\sqrt{n^2+\nu^2}$

BMN string is obtained for $n = 0$

open monodromy matrix and quasimomenta

BMN $\quad l_\sigma = -\frac{2x}{x^2-1}\nu(P_0 + P_8) \quad$ conts. $\qquad \Omega = -\frac{2\pi\nu x}{x^2-1}$

$$T(\pi,0,x) = \exp\Big(\Omega(P_0+P_8)\Big) \quad T_{0,\pi}(x) = (-)\begin{pmatrix} M & \\ & \mathrm{diag}(e^{i\Omega}, e^{i\Omega}, e^{-i\Omega}, e^{-i\Omega}) \end{pmatrix}$$

$$M = \begin{pmatrix} \cos\Omega & 0 & \sin\Omega & 0 \\ 0 & \cos\Omega & 0 & -\sin\Omega \\ -\sin\Omega & 0 & \cos\Omega & 0 \\ 0 & \sin\Omega & 0 & \cos\Omega \end{pmatrix} \qquad \hat{p}_{1,2} = -\hat{p}_{3,4} = \frac{2\pi\nu x}{x^2-1}$$

$$\tilde{p}_{1,2} = -\tilde{p}_{3,4} = \frac{2\pi\nu x}{x^2-1}$$

Quasimomenta for solutions with $n = 2N \neq 0$

open monodromy matrix built from L_σ instead of l_σ

matrix form of $L_\sigma = \begin{pmatrix} H & 0 \\ 0 & K \end{pmatrix}$

in the "AdS_5 corner" $K = -\frac{2x}{x^2-1}\kappa\frac{i}{2}\mathrm{diag}\,(1,1,-1,-1)$

AdS_5 quasimomenta $\hat{p}_{1,2} = -\hat{p}_{3,4} = \frac{2\pi\kappa x}{x^2-1} = \frac{x}{x^2-1}\frac{E}{g}$

H in the "S_5 corner" σ dependent solve diff. equ. to get monodromy

S^5 quasimomenta:

$\tilde{p}_1 = \frac{2\pi x}{x^2-1}\sqrt{\frac{n^2}{x^2}+w^2} = -\tilde{p}_4$ $\tilde{p}_2 = \frac{2\pi x}{x^2-1}\sqrt{n^2x^2+w^2} = -\tilde{p}_3$

reflection and inversion requirements residue synchronization satisfied

Quasiclassical fluctuations of open string solutions

algebraic curve (AC) efficient way for semiclassical contributions

follow procedure for closed string [Gromov]
add microscopic cuts poles to AC satisfying all requirements

for the BMN string $\hat{p}_{1,2} = -\hat{p}_{3,4} = \frac{2\pi\nu x}{x^2-1}$ $p(x) \to p(x) + \delta p(x)$

x_n^{ij} pole shared by sheets i j $p_i(x_n^{ij}) - p_j(x_n^{ij}) = 2\pi n$ $|x_n^{ij}| > 1$

$x_n^{ij} \to x_n = \frac{1}{n}(\nu + \sqrt{n^2 + \nu^2})$ N_n^{ij} excitation # $N^{ij} = \sum_n N_n^{ij}$

$p(x) \to p(x) + \delta p(x)$ analytic and satisfy

- poles at x_n^{ij} with residues $\pm\frac{1}{g}\alpha(x_n^{ij}) N_n^{ij}$ $\alpha(x) = \frac{x^2}{x^2-1}$

- obeying the $x \to 1/x$ and the $x \to -x$ symmetry properties

$$\tilde{p}_{1,2}(x) = -\tilde{p}_{2,1}(1/x) \qquad \tilde{p}_{3,4}(x) = -\tilde{p}_{4,3}(1/x) \qquad \tilde{p}_i(-x) = -\tilde{p}_i(x)$$
$$\hat{p}_{1,2}(x) = -\hat{p}_{2,1}(1/x) \qquad \hat{p}_{3,4}(x) = -\hat{p}_{4,3}(1/x) \qquad \hat{p}_i(-x) = -\hat{p}_i(x)$$

- residues at $x = \pm 1$ coincide for $\hat{p}_1, \hat{p}_2, \tilde{p}_1, \tilde{p}_2$ and for $\hat{p}_3, \hat{p}_4, \tilde{p}_3, \tilde{p}_4$

- the large x asymptotics of $\delta p(x)$ $\qquad \sum\limits_i \equiv \sum\limits_{i=\hat{3}\hat{4}\tilde{3}\tilde{4}} \qquad \sum\limits_k \equiv \sum\limits_{k=\hat{1}\hat{2}\tilde{1}\tilde{2}}$

$$\begin{pmatrix} \delta\hat{p}_1 \\ \delta\hat{p}_2 \\ \delta\hat{p}_3 \\ \delta\hat{p}_4 \end{pmatrix} \sim \frac{1}{xg} \begin{pmatrix} \delta\Delta + 2\sum\limits_i N^{\hat{1}i} \\ \delta\Delta + 2\sum\limits_i N^{\hat{2}i} \\ -\delta\Delta - 2\sum\limits_k N^{k\hat{3}} \\ -\delta\Delta - 2\sum\limits_k N^{k\hat{4}} \end{pmatrix} \qquad \begin{pmatrix} \delta\tilde{p}_1 \\ \delta\tilde{p}_2 \\ \delta\tilde{p}_3 \\ \delta\tilde{p}_4 \end{pmatrix} \sim \frac{1}{xg} \begin{pmatrix} -2\sum\limits_i N^{\tilde{1}i} \\ -2\sum\limits_i N^{\tilde{2}i} \\ 2\sum\limits_k N^{k\tilde{3}} \\ 2\sum\limits_k N^{k\tilde{4}} \end{pmatrix}$$

$\delta\Delta$ determines the energy $\qquad E = \delta\Delta + \text{excitation numbers}$

for the BMN δp_i explicitly

$$g \cdot \delta\hat{p}_2(x) = \hat{\alpha}\frac{2x}{x^2-1} + \sum_{i,n}\left(\frac{\alpha(x_n^{\hat{2}i})N_n^{\hat{2}i}}{x-x_n^{\hat{2}i}} + \frac{\alpha(x_n^{\hat{2}i})N_n^{\hat{2}i}}{x+x_n^{\hat{2}i}} + \frac{\alpha(x_n^{\hat{1}i})N_n^{\hat{1}i}}{1/x-x_n^{\hat{1}i}} + \frac{\alpha(x_n^{\hat{1}i})N_n^{\hat{1}i}}{1/x+x_n^{\hat{1}i}}\right)$$

$$g \cdot \delta\hat{p}_3(x) = \hat{\beta}\frac{2x}{x^2-1} + \sum_{k,n}\left(\frac{\alpha(x_n^{\hat{3}k})N_n^{\hat{3}k}}{x-x_n^{\hat{3}k}} + \frac{\alpha(x_n^{\hat{3}k})N_n^{\hat{3}k}}{x+x_n^{\hat{3}k}} + \frac{\alpha(x_n^{\hat{4}k})N_n^{\hat{4}k}}{1/x-x_n^{\hat{4}k}} + \frac{\alpha(x_n^{\hat{4}k})N_n^{\hat{4}k}}{1/x+x_n^{\hat{4}k}}\right)$$

$\delta\tilde{p}_2$: $\delta\hat{p}_2$ with $\hat{1}\hat{2} \to \tilde{1}\tilde{2}$ etc. $\delta\hat{p}_1(x) = -\delta\hat{p}_2(1/x)$ etc.

matching explicit δp_i to expected asymptotics equ. for $\delta\Delta$ $\hat{\alpha}$ $\hat{\beta}$

solution with no constraints on N_n^{ij} for closed string $\exists$ constraint

consistent solution $\delta\Delta = \sum_n \frac{\sqrt{\nu^2+n^2}-\nu}{\nu}\sum_i(N_n^{\hat{1}i} + N_n^{\hat{2}i} + N_n^{\tilde{1}i} + N_n^{\tilde{2}i})$

Quasimomenta from the Y system

closed string

[GKT] strong coupling scaling limit $g \sim L \sim N \to \infty$ of as. Y functions $\to$
$\to$ quasimomenta of closed string theory

[BNPS] $\quad Y = 0 \quad$ spectral problem by the same T and Y systems
asymptotic solution and analytic properties different

closed string/periodic	open string/ $Y = 0$
transfer matrix	double row transfer matrix
$Y_{a.0} = e^{-\tilde{\epsilon}_a L} t_{a,1} \dot{t}_{a,1}$	$Y_{a.0} = e^{-2\tilde{\epsilon}_a L} d_{a,1} \dot{d}_{a,1}$

$t_{a,1} \quad d_{a,1}$ solve the same T system

take strong coupling scaling limit of "new" $Y = 0$ Y functions
to obtain quasimomenta of open strings

fund. double row transfer m. $\rightarrow$ qu. generating function $\rightarrow$ scaling limit

$\mathcal{W}^{-1} = \sum_a (-1)^a \mathcal{D}^a \mathbb{D}_{a,1} \mathcal{D}^a$ $\quad \mathcal{D} = e^{-\frac{i}{2}\partial_u}$ thus $\mathcal{D} f(u) = f(u - \frac{i}{2})\mathcal{D}$

$SU(2)$ grading in $SU(2)$ subsector multiparticle states $1\dot{1}$ type

scaling limit $u = 2gz$ $\quad x^{\pm}(z) = x(z \pm \frac{i}{4g})$ $\quad \mathcal{D}$ only a formal expansion parameter assuming [GKT] relation between $\mathcal{D}$ and classical one

$$\hat{p}_1(x) = -\hat{p}_4(x) = -\hat{p}_2(1/x) = \hat{p}_3(1/x) = \frac{(J+2\mathcal{Q}_2)x}{g(x^2-1)} + B(x)$$

$$\tilde{p}_1(x) = -\tilde{p}_4(x) = -\tilde{p}_2(1/x) = \tilde{p}_3(1/x) =$$

$$= \frac{Jx}{g(x^2-1)} + B(x) + \frac{1}{g}(H_-(1/x) + H_+(1/x))$$

$J = L + 1 + N$ $\quad H_{\mp}(x) = \sum_{j=1}^{N} \frac{x^2}{x^2-1}\frac{1}{x \mp x_j}$ $\quad \mathcal{Q}_2 = \sum_j \frac{1}{x_j^2-1}$

$B(x) = \frac{1}{2g}\frac{x}{x^2+1}$ boundary contribution subleading

Generic states particles with labels $a\dot{b}$ [Galleas] [BNPS]

x_j $2m_1^L$ $(2m_1^R)$ y roots $2m_2^L$ $(2m_2^R)$ w roots $SU(2|2)_{L,R}$

procedure repeated succesfully

simple case $sl(2)$ sector (by duality)
N fundamental particles $3\dot{3}$ type

$$\hat{p}_1(x) = -\hat{p}_2(1/x) = -\hat{p}_4(x) = \hat{p}_3(1/x) =$$

$$= \frac{(L+1+2\sum_j \frac{1}{x_j^2-1})x}{g(x^2-1)} + B(x) + \frac{1}{g}\sum_{\epsilon=\pm} H_\epsilon(1/x)$$

$$\tilde{p}_1(x) = -\tilde{p}_2(1/x) = -\tilde{p}_4(x) = \tilde{p}_3(1/x) = \frac{(L+1)x}{g(x^2-1)} + B(x)$$

Conclusions

• described open monodromy matrix $T_{0,\pi}(x)$ and some of its properties (global charges, symmetry equations)

• provides framework for classical spectral curve of $Y = 0$ brane

• explicit quasimomenta quasiclassical fluctuations

• strong coupling scaling limit of asymptotic T and Y functions
$\rightarrow$ consistent expressions for quasimomenta

• work in progress: generalization to $O(N)$ sigma models with integrable boundaries